Pre-Calculus 10 Enriched Radicals

Lesson 1 Simplifying Radical Expressions

Recall:

index $\longrightarrow$ $\sqrt[n]{x}$ $\longleftarrow$ radicand

Recall: Multiplication Property of Radicals:

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$,

Where n is a natural number, and a and b are real numbers

Examples
Simplify the following radicals.

1. $\sqrt{50x^7}$

$x \geq 0$

$\sqrt{25 \cdot 2\, x^1 \cdot x^6}$

$\sqrt{25x^6}\,\sqrt{2x}$

$5x^3\sqrt{2x}$

Recall:
add exponents

$\underbrace{5x^3 \cdot 5x^3}_{\text{add exponents}}$

$25x^6$

$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$

2. $\sqrt[3]{24x^{10}y^3}$

exponents need to be divisible by 3

$\sqrt[3]{8 \cdot 3\, x^9 \cdot x^1\, y^3}$

$\sqrt[3]{8x^9y^3}\,\sqrt[3]{3x}$

$2x^3y\,\sqrt[3]{3x}$

$x \in \mathbb{R}$, $y \in \mathbb{R}$

x is an element of the real number set

$2^3 = 8$
$3^3 = 27$
$4^3 = 64$

Similarly to the Multiplication Property of Radicals, we can simplify radicals using the division property of radicals.

Equivalent expressions for any number have the same value.

- $\sqrt{\frac{16}{9}}$ is equivalent to $\frac{\sqrt{16}}{\sqrt{9}}$ because:

$$\sqrt{\frac{16}{9}} = \sqrt{\frac{4}{3} \cdot \frac{4}{3}} = \frac{4}{3} \quad \text{and} \quad \frac{\sqrt{16}}{\sqrt{9}} = \frac{\sqrt{4 \cdot 4}}{\sqrt{3 \cdot 3}} = \frac{4}{3}$$

A similar result is true for any index, n.

Division Property of Radicals:

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \text{where } n \in \mathbb{N} \text{ and } a, b, \sqrt[n]{a}, \sqrt[n]{b} \in \mathbb{R}, b \neq 0$$

Examples

3. Write $\sqrt[3]{-\frac{40}{81}}$ as a mixed radical.

$\frac{\sqrt[3]{-40}}{\sqrt[3]{81}}$ $\quad$ $\frac{\sqrt[3]{-8 \cdot 5}}{\sqrt[3]{27 \cdot 3}}$ $\quad$ $\frac{-2\sqrt[3]{5}}{3\sqrt[3]{3}}$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

or $-\frac{2}{3}\sqrt[3]{\frac{5}{3}}$

4. Write $-2\sqrt[3]{\frac{3}{4}}$ as an entire radical

$\sqrt[3]{\frac{(-2)^3 \cdot 3}{4}}$

$\sqrt[3]{\frac{-\cancel{8}^{2} \cdot 3}{\cancel{4}}}$

$\sqrt[3]{-6}$

Examples
Simply, if possible. State the permissible values of the variable.

5. $\sqrt{5a^2}$

$a \in \mathbb{R}$

"a" could be +ve or -ve and the root would still work
however, we never get a negative value from a principal square root

$|a|\sqrt{5}$

We use absolute value bars around variables to make sure a principal square root is always positive

ie $|2| = 2$ $|-2| = 2$

6. $\sqrt{-27b^5}$

$b \leq 0$

$\sqrt{-9 \cdot 3 b^4 b}$

$3b^2\sqrt{-3b}$

can't take the square root if sub in a positive value for b

7. $\sqrt[4]{7z}$

$z \geq 0$

8. $\sqrt[3]{24x^7}$

$x \in \mathbb{R}$

$\sqrt[3]{8 \cdot 3 x^6 x}$

$\sqrt[3]{8x^6}\ \sqrt[3]{3x}$

$2x^2\sqrt[3]{3x}$

can't take the cube root of a positive, 0 or negative value

worksheet
#2, 5, 9, 13, 16, 19, 22, 25, 26, 31, 34, 35, 40