Lesson 4 Converting to Standard Form - Completing the Square

Recall: Perfect Square Trinomials

Factor this trinomial

$x^2 - 10x + 25$

$(x-5)(x-5)$

or $(x-5)^2$

An equation of a quadratic function in ***general form*** is: $y = ax^2 + bx + c$

An equation of a quadratic function in ***standard form*** is: $y = a(x-h)^2 + k$

When the equation of a quadratic function is in general form, most characteristics of the graph cannot be identified. For this reason, it is useful to be able to convert from general form to standard form. This can be done by ***completing the square***.

Example 1
Convert $y = x^2 - 6x + 11$ to standard form.

$y = (x^2 - 6x \quad\quad) + 11$

Step 1: Bracket the "x" terms.

$\left(\frac{-6}{2}\right)^2$

$y = (\underbrace{x^2 - 6x + 9}_{\text{perfect square trinomial}}) + 11 - 9$

Step 2: Complete the square. Divide the coefficient of the "x" term by 2 and square it (This number will make a perfect square trinomial). Balance the equation.

$y = (x-3)(x-3) + 2$

$y = (x-3)^2 + 2$

Step 3: Factor the perfect square trinomial and rewrite as the square of a binomial.

Example 2
Convert $y = x^2 - 4x + 10$ to standard form.

$\left(\frac{-4}{2}\right)^2$

$y = (x^2 - 4x + 4) + 10 - 4$

$y = (x-2)^2 + 6$

half of (-4)

sum

Example 3
Convert $y = (2x^2 - 8x) - 7$ to standard form.

factor out

$\left(\frac{-4}{2}\right)^2$

$y = 2(x^2 - 4x + 4) - 7 - 2(4)$

$2(4)$ balance

$y = 2(x-2)^2 - 15$

Step 1: Bracket the "x" terms and factor out the numerical coefficient (include the sign)

Step 2: Complete the square. Divide the coefficient of the "x" term by 2 and square it (This number will make a perfect square trinomial). Balance the equation.

Step 3: Balance the equation (multiply the number added in step 2 by the coefficient)

Step 4: Factor the perfect square trinomial and rewrite as the square of a binomial.

Example 4

Write in standard form: $y = -\frac{1}{2}x^2 - 3x + 5$

$y = -\frac{1}{2}(x^2 + 6x + 9) + 5 + \frac{9}{2}$

$-\frac{1}{2}(9)$

$y = -\frac{1}{2}(x+3)^2 + \frac{10}{2} + \frac{9}{2}$

$y = -\frac{1}{2}(x+3)^2 + \frac{19}{2}$

Example 5

Identify the intercepts, the equation of the axis of symmetry, and the coordinates of the vertex of the graph of

a.) $y = 3x^2 - 12x + 7$

$y = 3(x^2 - 4x + 4) + 7 - 12$

$3(4)$

$y = 3(x-2)^2 - 5$

$V(2, -5)$

$x = 2$

x-int $y=0$

$0 = 3(x-2)^2 - 5$

$5 = 3(x-2)^2$

$\frac{5}{3} = (x-2)^2$

$\pm\sqrt{\frac{5}{3}} = x - 2$

exact values → $2 \pm \sqrt{\frac{5}{3}} = x$

y-int $x = 0$

$y = 3(0-2)^2 - 5$

$y = 3(4) - 5$

$y = 7$

b.) $y = -2x^2 + 10x - 3$

$\left(\frac{-5}{2}\right)^2$

$y = -2(x^2 - 5x + \frac{25}{4}) - 3 + \cancel{2}\left(\frac{25}{\cancel{4}_2}\right)$

$-2\left(\frac{25}{4}\right)$

$V\left(\frac{5}{2}, \frac{19}{2}\right)$

a.o.s. $x = \frac{5}{2}$

$y = -2(x - \frac{5}{2})^2 - 3 + \frac{25}{2}$

$y = -2(x - \frac{5}{2})^2 - \frac{6}{2} + \frac{25}{2}$

$y = -2(x - \frac{5}{2})^2 + \frac{19}{2}$

y-int $x=0$

$y = -2(0 - \frac{5}{2})^2 + \frac{19}{2}$

or

$y = -2(0)^2 + 10(0) - 3$

$y = -3$

x-int $y = 0$

or use quad formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-10 \pm \sqrt{10^2 - 4(-2)(-3)}}{2(-2)}$

$x = \frac{-10 \pm \sqrt{76}}{-4}$

$x = \frac{-10 \pm \sqrt{4 \cdot 19}}{-4}$

$x = \frac{-10 \pm 2\sqrt{19}}{-4}$

$x = \frac{5 \pm \sqrt{19}}{2}$

Exercise 4 Converting to Standard Form

1.) Convert to standard form: (follow example 1)
a.) $y = x^2 - 2x - 3$ b.) $y = x^2 + 6x + 5$
2.) Convert to standard form: (follow example 3)
a.) $y = 2x^2 + 5x + 2$ b.) $y = 2x^2 + 8x - 10$
3.) Convert to standard form: (follow example 5b)
a.) $y = -3x^2 + 18x + 1$ b.) $y = -x^2 - 3x + 5$
4.) Convert to standard form: (follow example 4)
$y = \frac{1}{2}x^2 - 5x - 3$
5.) Identify the intercepts, the equation of the axis of symmetry, and the coordinates of the vertex of the graph of: (follow example 5)
a.) $y = x^2 - 6x + 5$
b.) $y = -3(x + 1)^2$

Extra practice: Pg. 295 #3-10