Quadratic Functions: R3, R4

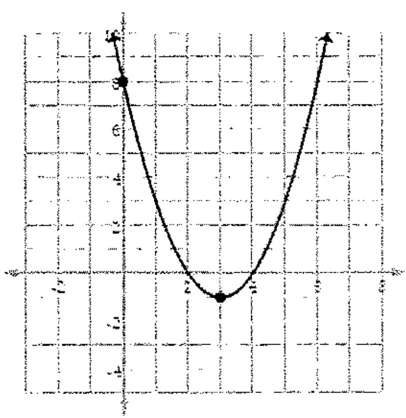
Characteristics:

- **Coordinates of the vertex**
- x**-intercept**
- y**-intercept**
- **Equation of the axis of symmetry**
- **Direction of opening**
- **Domain and range**

Vertex form: $y = a(x-h)^2 + k$
Factored form: $y = a(x-x_1)(x-x_2)$
General form: $y = ax^2 + bx + c$

***Note: Complete the square if equation is NOT in vertex form.**

1. Identify all characteristics:



Vertex: $(3,-1)$

Axis of Symmetry: $x = 3$

Max or Min @: min @ -1

y-intercept: 8

zero(s): 2, 4

Domain: $(-\infty, \infty)$

Range: $[-1, \infty)$

2. Identify all characteristics: $y = -\frac{1}{2}(x-4)^2 - 3$

y-int

$y = -\frac{1}{2}(0-4)^2 - 3$

$y = -\frac{1}{2}(16) - 3$

$y = -8 - 3$

$y = -11$

x-int

$0 = -\frac{1}{2}(x-4)^2 - 3$ (+3, +3)

$3 = -\frac{1}{2}(x-4)^2$ (×−2, ×−2)

$\sqrt{-6} = \sqrt{(x-4)^2}$

↓

no solution

Vertex: $(4,-3)$

Axis of Symmetry: $x = 4$

Max or Min @: max @ −3

y-intercept: −11

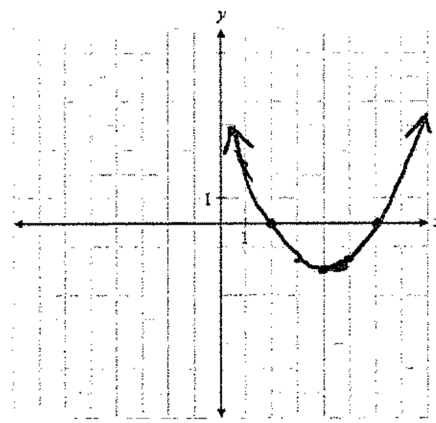
zero(s): none

Domain: $(-\infty, \infty)$

Range: $(-\infty, -3]$

3. Sketch the graph of $y = \frac{1}{2}(x-4)^2 - 2$

x	y	$\times\frac{1}{2}$
−2	4	2
−1	1	1/2
0	0	0
1	1	1/2
2	4	2



4. Complete the square: $y = (-2x^2 - 12x) + 4$

$y = -2(x^2 + 6x) + 4 \longrightarrow \left(\frac{6}{2}\right)^2$

$y = -2(x^2 + 6x + 9 - 9) + 4$

$y = -2(x^2 + 6x + 9) + 18 + 4$

$y = -2(x+3)^2 + 22$

5. Describe the transformations (from parent graph): $y = -5(x-6)^2 + 5$

- Reflection over x-axis
- Vertical Stretch by 5
- Horizontal Translation 6 right
- Vertical Translation 5 up

6. Write the equation of the quadratic function congruent to $y = 2x^2$, opening up, with intercepts of 3 and 5.

$$y = a(x - x_1)(x - x_2)$$
$$y = 2(x-3)(x-5)$$

7. Write the equation of the quadratic function that passes through the point $(3, -5)$, with a vertex of $(-3, 8)$.

$x \quad y$

$h \quad k$

$$y = a(x-h)^2 + k$$
$$-5 = a(3-(-3))^2 + 8$$
$$-5 = a(36) + 8$$
$$\frac{-13}{36} = \frac{36a}{36}$$
$$\frac{-13}{36} = a$$

$$\therefore y = \frac{-13}{36}(x+3)^2 + 8$$

8. Every week, a restaurant sells approximately 2 000 chicken wraps for $1.50 each. The manager determines that for every $0.10 increase in price, she will sell 100 fewer wraps.

a) What is the price of a wrap that will maximize the revenue?

$$R = (2000 - 100x)(1.50 + 0.1x)$$
$$= 3000 + 200x - 150x - 10x^2$$
$$= (-10x^2 + 50x) + 3000$$
$$= -10(x^2 - 5x) + 3000 \qquad \left(\frac{-5}{2}\right)^2 = 6.25$$
$$= -10(x^2 - 5x + 6.25 - 6.25) + 3000$$
$$= -10(x^2 - 5x + 6.25) + 62.5 + 3000$$
$$= -10(x - 2.5)^2 + 3062.50$$
$$(2.5, 3062.50)$$

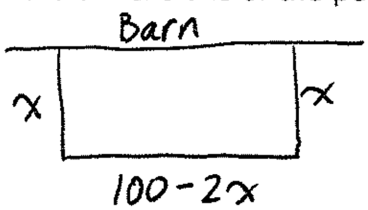
\# of inc/dec Max Rev

$$p = 1.50 + 0.1x$$
$$= 1.50 + 0.1(2.5)$$
$$= 1.50 + 0.25$$
$$\boxed{p = \$1.75}$$

b) What is the maximum revenue?

$3062.50

9. A rancher wants to build a rectangular pen, using one side of her barn for one side of the pen, and using 100m of fencing for the other three sides. What are the dimensions of the pen that has the largest area?



$$A_{max} = x(100-2x)$$
$$= 100x - 2x^2$$
$$= (-2x^2 + 100x)$$
$$= -2(x^2 - 50x) \longrightarrow \left(\frac{-50}{2}\right)^2 = 625$$
$$= -2(x^2 - 50x + 625 - 625)$$
$$= -2(x^2 - 50x + 625) + 1250$$
$$= -2(x - 25)^2 + 1250$$
$$(25, 1250)$$

Width ← 25, 1250 → max Area

$$\text{length} = 100 - 2(25)$$
$$= 100 - 50$$
$$= 50$$

∴ the dimensions are

25m × 50 m