Lesson 2 The Quadratic Formula

Completing the square can be used to solve quadratic equations that cannot be factored. The generalization to this solution is called the ***quadratic formula***. The quadratic formula is used to solve quadratic equations of the form $ax^2 + bx + c = 0, a \neq 0$.

Deriving the Quadratic Formula from $ax^2 + bx + c = 0$

1. Divide by "a" to make the coefficient of the x^2 term equal to 1.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

2. Isolate the terms with x.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

3. Complete the square and balance the equation.

$\frac{b}{a} \div 2$ $\quad \frac{b}{a} \cdot \frac{1}{2}$ $\quad \left(\frac{b}{2a}\right)^2$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

4. Simplify.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}\left(\frac{4a}{4a}\right)$$

5. Add right hand side using a Common Denominator.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

6. Take the square root of both sides.

$$\left(x + \frac{b}{2a}\right) = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

7. Simplify.

$$\left(x + \frac{b}{2a}\right) = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

8. Isolate x.

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

9. Combine fractions.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Quadratic Formula

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Example 1
Solve, using the quadratic formula.

a) $x^2 - 6x + 5 = 0$

$a = 1$
$b = -6$
$c = 5$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{16}}{2}$$

$$x = \frac{6 \pm 4}{2}$$

separate into two real roots

$$x = \frac{6+4}{2} \qquad x = \frac{6-4}{2}$$

$$x = 5 \qquad x = 1$$

factorable

$x^2 - 6x + 5 = 0$
$(x - 5)(x - 1) = 0$
$x = 5 \quad x = 1$

b) $x^2 - 6x = -7$

must be in the form $ax^2+bx+c=0$

$x^2 - 6x + 7 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{8}}{2}$$ ← complex, change to mixed

$\sqrt{8}$
$\sqrt{4 \cdot 2}$
$2\sqrt{2}$

$$x = \frac{6 \pm 2\sqrt{2}}{2}$$

$$x = \frac{6}{2} \pm \frac{2\sqrt{2}}{2}$$

$x = 3 \pm \sqrt{2}$

c) $3x^2 - 5x + 7 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(7)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{-59}}{6}$$ ← can't square root a -ve value

$\emptyset$

Example 2
Simplify.

a.) $x = \frac{5 \pm \sqrt{50}}{10}$

$$x = \frac{5 \pm 5\sqrt{2}}{10}$$

$$x = \frac{5}{10} \pm \frac{5\sqrt{2}}{10}$$

$$x = \frac{1}{2} \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{1 \pm \sqrt{2}}{2}$$

b.) $x = \frac{8 \pm \sqrt{24}}{4}$

$$x = \frac{8 \pm 2\sqrt{6}}{4}$$

$$x = \frac{4 \pm \sqrt{6}}{2}$$

Note:

$$x = \frac{8 \pm \sqrt{6}}{4}$$

does not reduce!!

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#1 a, b
3 a, b, c, d, f

Exercise 5 The Quadratic Formula

1.) Solve: (follow example 1)

a.) $3x^2 + 5x - 2 = 0$

b.) $-2x^2 + 3x + 8 = 0$

c.) $4x^2 - 12x = 9$

d.) $3x^2 = -5x + 1$

e.) $2x^2 + 4x + 7 = 0$

f.) $16x^2 + 24x = -9$

2.) Solve, using an appropriate method: (L3, L4, L5)

a.) $x^2 + 2x - 2 = 0$

b.) $-x^2 + 6x - 9 = 0$

c.) $-2x^2 + 16 = 0$

d.) $\frac{x^2}{2} - \frac{x}{2} = 1$

e.) $x^2 - 4x + 8 = 0$

Textbook: Pg. 218 #5a, c, 6b, d, 7a, b, 8