L8 Solving Quadratic Eqns by Factoring

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Lesson 8 Solving Quadratic Equations Using Factoring________

A quadratic equation is any equation which can be written in the form $ax^2 + bx + c = 0$ where $a \neq 0$ and a, b, and c are constants.

ie. $x^2 - 5x - 45 = 0$ contains a ***quadratic*** or ***second-degree*** term (a term with a variable that is squared) and no term of higher degree. This is an example of a quadratic ***equation*** because it only has one variable and contains an equals sign.
Note: $x^2 - 5x - 45$ is a ***quadratic expression.***

Solving Quadratic Equations

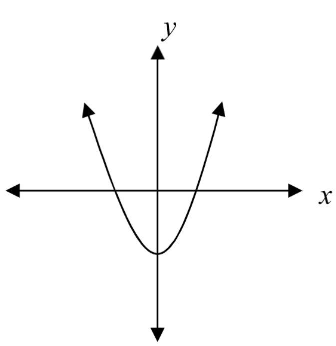
One strategy is to solve using factoring and the zero product property.

Zero product property: If the product of two factors is 0, then one or both factors must be equal to 0.
If $ab = 0$, then either $a = 0$ and/or $b = 0$.
It follows that if $(x + b)(x + d) = 0$, then $(x + b) = 0$, and/or $(x + d) = 0$

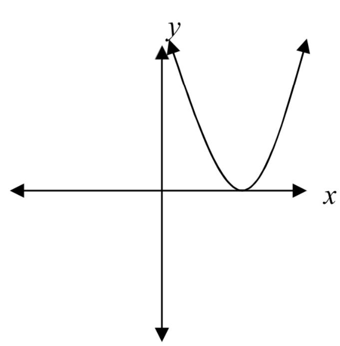
The solutions to a quadratic equation are called the ***roots*** (values which make the equation true) of the equation. The roots of a quadratic equation are the same values as the x-intercepts of the graph of $y = ax^2 + bx + c$, or the ***zeros*** of the corresponding quadratic function, $y = ax^2 + bx + c$.

Types of Solutions

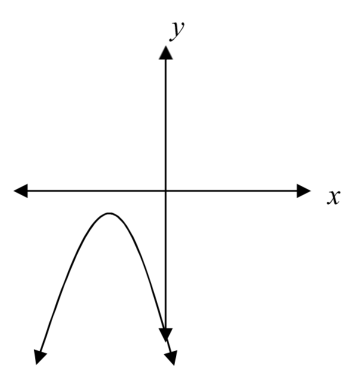
2 Solutions



1 Solution



0 Solutions



Example 1: Solving by Factoring
Solve each equation, then verify the solution.

a.) $(3x + 1)(x - 6) = 0$

Set each factor equal to 0 and solve for x

$3x + 1 = 0$ $\quad$ $x - 6 = 0$

$3x = -1$ $\quad$ $x = 6$

$x = -\frac{1}{3}$

b.) $x^2 - x - 56 = 0$

factor

$(x-8)(x+7) = 0$

$x - 8 = 0$ $\quad$ $x + 7 = 0$

$x = 8$ $\quad$ $x = -7$

Example 2
Solve, by factoring.

a.) $\frac{3x^2}{3} + \frac{75}{3} = -\frac{30x}{3}$

$x^2 + 25 = -10x$

One side of the eqn must be 0

$x^2 + 10x + 25 = 0$

$(x+5)^2 = 0$

$x + 5 = 0$

$x = -5$

b.) $\frac{5x^2}{5} = \frac{-20x}{5}$

Never divide by a variable because could be dividing by 0 unintentionally (* can't divide by 0)

$x^2 = -4x$

$x^2 + 4x = 0$

GCF x

$x(x+4) = 0$

$x = 0 \quad x = -4$ ← roots of the eqn ie values that make the eqn true

c.) $x(3x - 20) = -12$

$3x^2 - 20x + 12 = 0$

P 36
S −20
F $\frac{-18}{3}, \frac{-2}{1}$
−6 −2

$(3x - 2)(x - 6) = 0$

$3x - 2 = 0 \qquad x - 6 = 0$

$3x = 2 \qquad x = 6$

$x = \frac{2}{3}$

Example 3: Using Quadratic Equations to Solve Word Problems

a.) The sum of a number and its square is 20. Determine the number.

add, equals

$x + x^2 = 20$

$x^2 + x - 20 = 0$

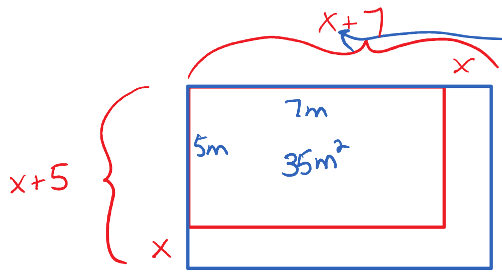
$(x+5)(x-4) = 0$

$x = -5 \quad x = 4$

∴ The number is either −5 or 4.

b.) A rectangular garden has dimensions 5m by 7m. When both dimensions are increased by the same length, the area of the garden increases by 45 m^2. Determine the dimensions of the larger garden.

new area
$35 + 45$
$80 m^2$



$A = l \cdot w$

$80 = (x+7)(x+5)$

$80 = x^2 + 5x + 7x + 35$

$0 = x^2 + 12x - 45$

$0 = (x+15)(x-3)$

$\cancel{x = -15}$ $\boxed{x = 3}$

length can't be -ve

Amount of increase is 3m

New Dimensions are
8m x 10m

Example 4: Determining Equations

Determine a quadratic equation which has roots of – 4 and $\frac{5}{3}$.

$x = -4$ $\quad x = \frac{5}{3}$

$x + 4 = 0$ $\quad 3x = 5$

$3x - 5 = 0$

$(x+4)(3x-5) = 0$

or

$3x^2 + 7x - 20 = 0$

☺ Make sure you know the difference between

Factor: $x^2 + x - 6$ and Solve $x^2 + x - 6 = 0$

Solution: $(x + 3)(x - 2)$ *Solution*: $x = -3$ and $x = 2$

☺ The RHS must be equal to 0 (or the zero product property does not apply).

☺ Make sure your solutions are logical for word problems.
ie. length can't be negative.

pg. 267
6a, c, i, j, m
7c, 9, j
8a, b

pg 270
10, 12, 14