QEQFII L4 The Discriminant

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Lesson 4 Interpreting the Discriminant

Recall: The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

← the discriminant (the expression under the radical)

The discriminant is used to determine the nature/characteristic of the roots of a quadratic equation. A quadratic equation can have:

- no real roots
- exactly one real root
- two real roots
- rational/irrational roots

Number of Roots of a Quadratic Equation

Given a quadratic equation, $ax^2 + bx + c = 0$,

- If the discriminant is positive ie. $b^2 - 4ac > 0$, then two real roots exist.
- If the discriminant is 0 ie. $b^2 - 4ac = 0$, then exactly one real root exists.
- If the discriminant is negative ie, $b^2 - 4ac < 0$, then no real roots exist.
- If the discriminant is 0 or a perfect square, then these roots are called rational roots.

$x = \frac{-4 \pm \sqrt{0}}{2}$

$x = \frac{-4+0}{2}$ $x = \frac{-4-0}{2}$

$x = -2$ $x = -2$

$x = \frac{-3 \pm \sqrt{-11}}{2}$

can't square root a -ve

non-perfect square → irrational roots

Example 1

Determine the nature of the roots of the quadratic equation, $9x^2 - 6x + 1 = 0$.

$\Delta = b^2 - 4ac$

$= (-6)^2 - 4(9)(1)$

$= 0$

$a = 9$
$b = -6$
$c = 1$

one rational root

Example 2
Given the following discriminant values, determine the characteristic of the roots.

a.) $b^2 - 4ac = 20$

↑ positive, non-perfect square

2 irrational roots

b.) $b^2 - 4ac = -42$

↑ negative

no real roots
(two different imaginary numbers)

c.) $b^2 - 4ac = 49$

↑ positive, perfect square

2 rational roots

Example 3
Determine the value(s) of k for which $2x^2 + 7x + k = 0$ has no real roots.

↳ negative discriminant

$a = 2$
$b = 7$
$c = k$

$b^2 - 4ac < 0$
$7^2 - 4(2)k < 0$
$7^2 - 8k < 0$
$49 < 8k$
$\frac{49}{8} < k$

OR

$7^2 - 8k < 0$
$-8k < -49$
$k > \frac{49}{8}$

↑ flip inequality when dividing or multiplying by a negative value

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2g, 3d
4b, c
5e