Lesson 7 Interpreting the Discriminant

Recall: The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ the discriminant (the expression under the radical)

The discriminant is used to determine the nature/characteristic of the roots of a quadratic equation. A quadratic equation can have:

- no real roots
- exactly one real root
- two real roots
- rational/irrational roots

Number of Roots of a Quadratic Equation

The quadratic equation $ax^2 + bx + c = 0$ has:

- If the discriminant is positive ie. $b^2 - 4ac > 0$, then two real roots exist. positive
- If the discriminant is 0 ie. $b^2 - 4ac = 0$, then exactly one real root exists.
- If the discriminant is negative ie, $b^2 - 4ac < 0$, then no real roots exist. negative
- If the discriminant is 0 or a perfect square, then these roots are called rational roots.

Example 1
Determine the nature of the roots of the quadratic equation, $9x^2 - 6x + 1 = 0$.

$\Delta = b^2 - 4ac$ a = 9

$\Delta = (-6)^2 - 4(9)(1)$ b = -6

$\Delta = 0$ c = 1

symbol for discriminant

∴ one real root exists and it is rational

Example 2
Given the following discriminant values, determine the characteristic of the roots.

a.) $b^2 - 4ac = 20$ ← discriminant

$b^2 - 4ac > 0$

$\therefore$ two real roots exist and they are irrational (since 20 is not a perfect square)

b.) $b^2 - 4ac = -42$

$b^2 - 4ac < 0$

$\therefore$ no real roots exist

since $x = \frac{-b \pm \sqrt{-42}}{2a}$

can't square root a −ve value

c.) $b^2 - 4ac = 49$

$b^2 - 4ac > 0$

$\therefore$ two real roots exist and they are rational (since 49 is a perfect square)

Example 3
Determine the values of k for which $2x^2 + 7x + k = 0$ has no real roots.

discriminant is −ve

$$b^2 - 4ac < 0$$
$$7^2 - 4(2)k < 0$$
$$49 - 8k < 0$$
$$\frac{-8k}{-8} < \frac{-49}{-8}$$
$$k > \frac{49}{8}$$

dividing by a −ve so inequality must flip

or

$$49 - 8k < 0$$
$$49 < 8k$$
$$\frac{49}{8} < k$$

Exercise 7 Interpreting the Discriminant

1.) Determine the nature of the roots of the quadratic equations: (follow example 1)

a.) $-2x^2 + 3x + 8 = 0$ 2 real roots, irrational

b.) $3x^2 - 5x = -9$ no real roots

c.) $\frac{1}{4}x^2 - 3x + 9 = 0$ one real root, rational

2.) Given the following discriminant values, determine the characteristic of the roots. (follow example 2)

a.) $b^2 - 4ac = -58$ no real roots

b.) $b^2 - 4ac = 144$ 2 real roots, rational

c.) $b^2 - 4ac = 12$ 2 real roots

3.) Determine the values of k for which $2x^2 - 3x + k = 0$ has exactly one real root. (follow example 3)

$k = \frac{9}{8}$

4.) Solve by completing the square: (follow L4, ex 2)

a.) $x^2 - 6x = 5$

$x = 3 \pm \sqrt{14}$

b.) $x^2 + 16x - 9 = 0$

$x = -8 \pm \sqrt{73}$

5.) Solve using square root principle (express answers as exact values): (follow L4, ex 1)

a.) $3x^2 - 8 = 12$

$x = \pm\sqrt{\frac{20}{3}}$

or

$\pm \frac{2\sqrt{5}}{\sqrt{3}}$

b.) $(x + 3)^2 = 17$

$x = -3 \pm \sqrt{17}$

Assignment: Pg. 232 #4-12