Pre-Calculus 11 Quadratic Equations

Lesson 5 The Quadratic Formula

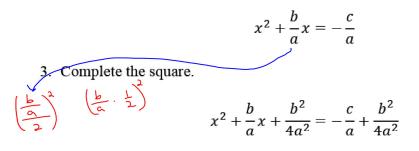
Completing the square can be used to solve quadratics that cannot be factored. The generalization to this solution is called the *quadratic formula*. The quadratic formula is used to solve quadratic equations of the form $ax^2 + bx + c = 0$, $a \neq 0$.

Deriving the Quadratic Formula from $ax^2 + bx + c = 0$

1. Divide by "a" so coefficient of x^2 is 1.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

2. Isolate the terms with x.



4. Simplify.

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}\left(\frac{4a}{4a}\right)$$

5. Find Common Denominator for right hand side.

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

6. Square root both sides.

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$$\left(x+\frac{b}{2a}\right) = \pm \sqrt{\frac{b^2-4ac}{4a^2}}$$

7. Simplify.

$$\left(x+\frac{b}{2a}\right) = \pm \frac{\sqrt{b^2-4ac}}{2a}$$

8. Isolate *x*.

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

9. Combine fractions.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \qquad -\int_{ax^2 + bx + c} = 0$$

Example 1

Solve, using the quadratic formula:

a)
$$x^2 - 6x + 5 = 0$$

a: 1 $x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$
b: -6 $x = -\frac{(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$
 $x = \frac{(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$
 $x = \frac{(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$
 $x = 5 \quad x = 1$
 $x = 5 \quad x = 1$
 $x = \frac{6 \pm 4}{2}$
 $x = \frac{6 \pm 4}{2}$
 $x = 5 \quad x = 1$

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b)
$$x^2 - 6x = -7$$

 $x^2 - 6x + 7 = 0$

 $x = -\frac{(-6) \pm \sqrt{(-6)^2 - 4(1)(1)}}{2(1)}$

 $x = \frac{6 \pm \sqrt{8}}{2}$

 $x = \frac{3\sqrt{2}}{2}$

 $x = \frac{5 \pm \sqrt{5}^2 - 4(3)(7)}{2(3)}$

 $x = \frac{5 \pm \sqrt{-57}}{6}$

 $x = \frac{5 \pm$

Example 2
Simplify
a.)
$$x = \frac{5 \pm \sqrt{50}}{10}$$

 $x = \frac{5 \pm \sqrt{24}}{4}$
 $x = \frac{5 \pm \sqrt{25 \cdot 2}}{10}$
 $x = \frac{5 \pm \sqrt{25 \cdot 2}}{10}$
 $x = \frac{8 \pm \sqrt{24}}{4}$
 $x = \frac{8 \pm \sqrt{24}}{4}$
 $x = \frac{8 \pm \sqrt{4 \cdot 6}}{4}$
 $x = \frac{8 \pm \sqrt{4} \cdot 6}{4}$
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 $x = \frac{8 \pm \sqrt{4} \cdot 6}{4}$
 $x = \frac{4 \pm \sqrt{6}}{4}$

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Exercise 5 The Quadratic Formula

1.) Solve: (follow example 1) a.) $3x^{2} + 5x - 2 = 0$ a) $x = \frac{1}{3}$ x = -2b) $-2x^{2} + 3x + 8 = 0$ b) $x = -\frac{3 \pm \sqrt{73}}{-4}$ c) $x = \frac{3 \pm 3\sqrt{2}}{2}$ c) $4x^{2} - 12x = 9$ d.) $3x^{2} = -5x + 1$ a) $x = -5 \pm \frac{\sqrt{33}}{6}$ e) $2x^{2} + 4x + 7 = 0$ f.) $16x^{2} + 24x = -9$ c) φ f.) $x = -\frac{3}{4}$ 2.) Solve, using an appropriate method: (L3, L4, L5) a.) $x^{2} + 2x - 2 = 0$ $x = -1 \pm \sqrt{3}$ b.) $-x^{2} + 6x - 9 = 0$ x = 3c.) $-2x^{2} + 16 = 0$ $x = \pm 2\sqrt{3}$ d.) $\frac{x^{2}}{2} - \frac{x}{2} = 1$ x = 2 + x = -3e.) $x^{2} - 4x + 8 = 0$ φ 1f) $\frac{1}{6x^{2} + 24x + 9}{2(16)}$ or factor $x = -\frac{24 \pm \sqrt{34x^{2} - 4(16)}(4)}{2(16)}$ (4x + 3)(4x + 3) = 0 $x = -\frac{24}{32}$ (4x + 3)(4x + 3) = 0 $\frac{4x + 3 = 0}{4x + 3 = 0}$

Extra Practice: Pg. 218 #5, 6, 7a, b, 8