Lesson 5 The Quadratic Formula

Completing the square can be used to solve quadratics that cannot be factored. The generalization to this solution is called the ***quadratic formula***. The quadratic formula is used to solve quadratic equations of the form $ax^2 + bx + c = 0, a \neq 0$.

Deriving the Quadratic Formula from $ax^2 + bx + c = 0$

1. Divide by "a" so coefficient of x^2 is 1.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

2. Isolate the terms with x.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

3. Complete the square.

$\left(\frac{\frac{b}{a}}{2}\right)^2$ $\left(\frac{b}{a} \cdot \frac{1}{2}\right)^2$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

4. Simplify.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}\left(\frac{4a}{4a}\right)$$

5. Find Common Denominator for right hand side.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

6. Square root both sides.

$$\left(x+\frac{b}{2a}\right)=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

7. Simplify.

$$\left(x+\frac{b}{2a}\right)=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

8. Isolate x.

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

9. Combine fractions.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

<u>The Quadratic Formula</u>

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

for $ax^2+bx+c=0$

Example 1
Solve, using the quadratic formula:

a) $x^2-6x+5=0$

$a=1$
$b=-6$
$c=5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(5)}}{2(1)}$$

$$x=\frac{6\pm\sqrt{16}}{2}$$

$$x=\frac{6\pm4}{2}$$

$$x=\frac{6+4}{2} \qquad x=\frac{6-4}{2}$$

$$x=5 \qquad x=1$$

or factor

$(x-5)(x-1)=0$

$x=5 \quad x=1$

b) $x^2 - 6x = -7$

$x^2 - 6x + 7 = 0$ ← must be 0

$a = 1$
$b = -6$
$c = 7$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{8}}{2}$$

$\sqrt{8}$
$\sqrt{4 \cdot 2}$
$2\sqrt{2}$

$$x = \frac{6 \pm 2\sqrt{2}}{2}$$

$$x = \frac{6}{2} \pm \frac{2\sqrt{2}}{2}$$

$$x = 3 \pm \sqrt{2}$$

c) $3x^2 - 5x + 7 = 0$

$a = 3$
$b = -5$
$c = 7$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(7)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{-59}}{6}$$ ← cannot square root a negative value

$\therefore$ no sol'n $\emptyset$

no real roots exist

Example 2
Simplify

a.) $x = \frac{5 \pm \sqrt{50}}{10}$

$$x = \frac{5 \pm \sqrt{25 \cdot 2}}{10}$$

$$x = \frac{5 \pm 5\sqrt{2}}{10}$$ $\div$ each term by 5

$$x = \frac{1 \pm \sqrt{2}}{2}$$

b.) $x = \frac{8 \pm \sqrt{24}}{4}$

$$x = \frac{8 \pm \sqrt{4 \cdot 6}}{4}$$

$$x = \frac{8 \pm 2\sqrt{6}}{4}$$

$$x = \frac{4 \pm \sqrt{6}}{2}$$

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Exercise 5 The Quadratic Formula

1.) Solve: (follow example 1)
a.) $3x^2 + 5x - 2 = 0$
b.) $-2x^2 + 3x + 8 = 0$
c.) $4x^2 - 12x = 9$
d.) $3x^2 = -5x + 1$
e.) $2x^2 + 4x + 7 = 0$
f.) $16x^2 + 24x = -9$

a) $x = \frac{1}{3}$ $x = -2$
b) $x = \frac{-3 \pm \sqrt{73}}{-4}$
c) $x = \frac{3 \pm 3\sqrt{2}}{2}$
d) $x = \frac{-5 \pm \sqrt{37}}{6}$
e) $\emptyset$
f) $x = -\frac{3}{4}$

2.) Solve, using an appropriate method: (L3, L4, L5)
a.) $x^2 + 2x - 2 = 0$ $x = -1 \pm \sqrt{3}$
b.) $-x^2 + 6x - 9 = 0$ $x = 3$
c.) $-2x^2 + 16 = 0$ $x = \pm 2\sqrt{2}$
d.) $\frac{x^2}{2} - \frac{x}{2} = 1$ $x = 2$ $x = -1$
e.) $x^2 - 4x + 8 = 0$ $\emptyset$

1f) $16x^2 + 24x + 9 = 0$ $\longrightarrow$ or factor perfect square trinomial

$$x = \frac{-24 \pm \sqrt{\cancel{24^2 - 4(16)(9)}}^{\,0}}{2(16)}$$

$$x = \frac{-24}{32}$$

$$x = -\frac{3}{4}$$

$$(4x + 3)(4x + 3) = 0$$

$$4x + 3 = 0$$

$$4x = -3$$

$$x = -\frac{3}{4}$$

Extra Practice: Pg. 218 #5, 6, 7a, b, 8