Pre-Calculus 11 Quadratic Equations

Lesson 4 Using Square Roots to Solve Quadratic Equations

Solving Quadratic Equations Using the Square Root Principle

When $b = 0$, the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, becomes $ax^2 + c = 0$. If this equation has a solution, it can be solved using square roots.

Example 1
Solve each equation. Verify the solution.

a) $3x^2 \cancel{- 7} = 8$ $\cancel{+7}$ $+7$

$\frac{\cancel{3}x^2}{\cancel{3}} = \frac{15}{3}$

$x^2 = 5$

$x = \pm\sqrt{5}$ ← simple radical

$\pm\sqrt{5}$ because

$(\sqrt{5})(\sqrt{5}) = 5$

$(-\sqrt{5})(-\sqrt{5}) = 5$

b) $(x + 3)^2 = 20$

$x + 3 = \pm\sqrt{20}$

$x = -3 \pm \sqrt{20}$ ← complex radical

$x = -3 \pm \sqrt{4 \cdot 5}$ ← largest perfect square factor of 20

$x = -3 \pm 2\sqrt{5}$ ← exact values

c) $3x^2 + 12 = 0$

$3x^2 = -12$

$x^2 = -4$ ← can't square root a -ve value

$\emptyset$ ← empty set

or

No solution

Completing the Square

Recall: Perfect Square Trinomials.

Factor: $x^2 + 6x + 9$

P 9
S 6
F 3, 3

$(x+3)(x+3)$

or

$(x+3)^2$

$x^2 - 4x + 4$

P 4
S -4
F -2, -2

$(x-2)(x-2)$

or

$(x-2)^2$

Example 2
Solve, by completing the square

a.) $x^2 + 6x = 16$

divide "b" by 2 and square

to make a perfect square trinomial

$\left(\frac{6}{2}\right)^2$

$x^2 + 6x + 9 = 16 + 9$ ← balance (what we do to one side, we must do to the other)

PSF → $(x+3)(x+3) = 25$

$(x+3)^2 = 25$

$x+3 = \pm 5$

$x = -3 \pm 5$

split into two roots

$x = -3 + 5$ $\qquad$ $x = -3 - 5$

$x = 2$ $\qquad$ $x = -8$

b.) $x^2 + 8x - 10 = 0$

$\left(\frac{8}{2}\right)^2$

$x^2 + 8x + 16 = 10 + 16$

$(x+4)(x+4) = 26$

$(x+4)^2 = 26$

$x + 4 = \pm\sqrt{26}$

$x = -4 \pm \sqrt{26}$

Steps

1. Get terms with x on one side
2. Complete the square (divide "b" by 2 and square)
3. Balance the equation (add new value to both sides)
4. Factor perfect square trinomial
5. Write as squared factor
6. Square root both sides (don't forget ±)
7. Isolate x

together

c.) $\left(-\frac{1}{2}x^2 + 6x - 1\right) = 0$ $\times(-2)$

Multiply both sides by -2

$x^2 - 12x + 2 = 0$

$\left(\frac{-12}{2}\right)^2$

$x^2 - 12x + 36 = -2 + 36$

$(x-6)(x-6) = 34$ ⟵ do not need to show

$(x-6)^2 = 34$

$x - 6 = \pm\sqrt{34}$

$x = 6 \pm \sqrt{34}$

Example 3

A football is kicked vertically. The approximate height of the football, h metres, after t seconds is modelled by this formula: $h = 1 + 20t - 5t^2$. Determine when ~~will~~ the football will hit the ground (to the nearest tenth of a second).

$h = 0$

$h = 1 + 20t - 5t^2$

$0 = 1 + 20t - 5t^2$

$5t^2 - 20t = 1$

$5(t^2 - 4t + 4) = 1 + 20$

$5(t-2)^2 = 21$

$(t-2)^2 = \frac{21}{5}$

$t - 2 = \pm\sqrt{\frac{21}{5}}$

$t = 2 \pm \sqrt{\frac{21}{5}}$

$t = 2 + \sqrt{\left(\frac{21}{5}\right)}$ $t = 2 - \sqrt{\left(\frac{21}{5}\right)}$

$t = 4.0$ t $~~-0.05~~$

It will hit the ground after 4.0 seconds.

rej time can't be -ve

Exercise 4 Using Square Roots to Solve Quadratic Equations

1.) Solve (express answers as exact values): (follow example 1)

a.) $2x^2 - 8 = 0$ $x = \pm 2$ b.) $(x+2)^2 = 7$ $x = -2 \pm \sqrt{7}$

2.) Solve: (follow example 2)

a.) $x^2 - 8x = 4$ $x = 4 \pm 2\sqrt{5}$ b.) $x^2 + 10x + 4 = 0$ $x = -5 \pm \sqrt{21}$

c.) $\frac{1}{2}x^2 - 6x - 5 = 0$ $x = 6 \pm \sqrt{46}$

3.) Word Problem (follow example 3)

The path of debris from fireworks when the wind is about 25 km/h can be modelled by the quadratic function $h = -\frac{1}{2}x^2 + x + 7$ where h is the height and x is the horizontal distance travelled, in metres. Determine how far away from the launch site the debris will land, to the nearest tenth of a metre.

4.9 m

3)

$0 = -\frac{1}{2}x^2 + x + 7$

$0 = x^2 - 2x - 14$ $\left(-\frac{2}{2}\right)^2$

$1 + 14 = x^2 - 2x + 1$

$15 = (x-1)^2$

$\pm\sqrt{15} = x - 1$

$1 \pm \sqrt{15} = x$

$4.9\text{m} = x$ $\cancel{x = -2.9}$

Extra Practice: Pg. ~~206 #4, 5a,b, 8, 9, 10 b, 11a, 12, 13~~

pg 206 #5a,b, 8a,b, 9, 10b, 12

2b) $x^2 + 10x + 4 = 0$

$x^2 + 10x + 25 = -4 + 25$

$(x+5)^2 = 21$

$x + 5 = \pm\sqrt{21}$

$x = -5 \pm \sqrt{21}$

2 a) $x^2 - 8x = 4$

$x^2 - 8x + 16 = 4 + 16$

$(x-4)^2 = 20$

$x - 4 = \pm\sqrt{20}$

$x = 4 \pm \sqrt{20}$

$x = 4 \pm 2\sqrt{5}$