Lesson 3 Solving Quadratic Equations by Factoring

A quadratic equation is any equation which can be written in the form $ax^2 + bx + c = 0$ where $a \neq 0$ and a, b, and c are constants.

ie. $x^2 - 5x - 45 = 0$ contains a ***quadratic*** or ***second-degree*** term (a term with a variable that is squared) and no term of higher degree. This is an example of a ***quadratic equation*** because it has an equals sign.
Note: $x^2 - 5x - 45$ is a ***quadratic expression***

Solving Quadratic Equations

One strategy is to solve by factoring and apply the zero product property.

Zero product property: If the product of two numbers is 0, then one or both numbers must be equal to 0.

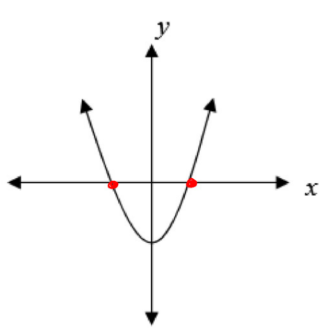
ie. $(x + b)(x + d) = 0$, then $(x + b) = 0$, and/or $(x + d) = 0$

The solutions to a quadratic equation are called the ***roots*** (values which make the equation true) of the equation. The roots of a quadratic equation are the same values as the x-intercepts of the graph of $y = ax^2 + bx + c$, or the ***zeros*** of the corresponding quadratic function, $y = ax^2 + bx + c$.

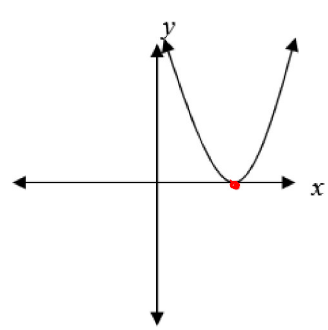
Types of Solutions $y = ax^2 + bx + c \Rightarrow 0 = ax^2 + bx + c$

y = 0 on the x-axis

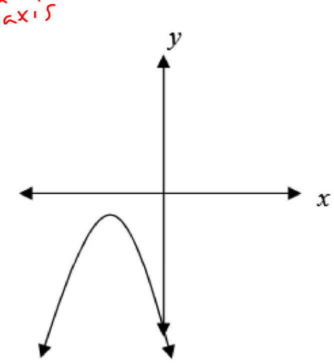
2 Solutions



1 Solution



0 Solutions



Example 1: Solve by Factoring

Solve each equation, then verify the solution.

a) $x^2 - x - 56 = 0$

P -56
S -1
F -8, 7

$(x-8)(x+7) = 0$

$x - 8 = 0$ $x + 7 = 0$

$x = 8$ $x = -7$

① Make sure one side equals 0

② Factor

③ Set each factor equal to 0 (zero product principle)

④ Solve

b) $(3x + 1)(x - 6) = 0$

$3x + 1 = 0$ $x - 6 = 0$

$3x = -1$ $x = 6$

$x = -\frac{1}{3}$

Already in factored form and RHS equals 0

Example 2

Solve, by factoring.

a.) $3x^2 + 75 \overset{+30x}{=} -30x^{+30x}$

$$\frac{3x^2}{3} + \frac{30x}{3} + \frac{75}{3} = \frac{0}{3}$$

one side must be 0

GCF 3
since it's an equation,
we divide both sides by 3

$x^2 + 10x + 25 = 0$

$(x+5)(x+5) = 0$

perfect square trinomial
2 identical factors

$x + 5 = 0$

$x = -5$

b.) $5x^2 = -20x$

$5x^2 + 20x = 0$

$x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0 \qquad x + 4 = 0$

$x = -4$

GCF 5 (of 5 and 20)
divide both sides by 5

* Never divide by a variable!!

or

$5x(x+4) = 0$

$5x = 0 \qquad x = -4$

$x = 0$

Example 3: Using Quadratic Equations to Solve Word Problems

a.) The sum of a number and its square is 20. Determine the number.

add x x^2

$$x + x^2 = 20$$

must be 0

$$x^2 + x - 20 = 0$$

P -20
S 1
F 5, -4

$$(x+5)(x-4) = 0$$

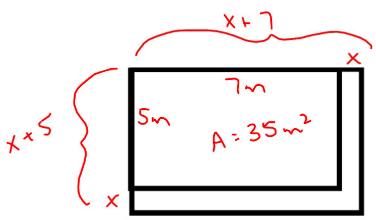
zero product principle
set each factor = 0
and solve

$$x + 5 = 0 \qquad x - 4 = 0$$

$$x = -5 \qquad x = 4$$

∴ the number is either -5 or 4

b.) A rectangular garden has dimensions 5m by 7m. When both dimensions are increased by the same length, the area of the garden increases by 45 m^2. Determine the dimensions of the larger garden.



new area
$35 + 45 = 80\text{m}^2$

$$A = l \cdot w$$

$$\underset{-80}{80} = \underbrace{(x+7)(x+5)}_{\text{FOIL}} \; -80 \quad \text{(multiply)}$$

$$0 = x^2 + 5x + 7x + 35 - 80$$

$$0 = x^2 + 12x - 45$$

$$0 = (x+15)(x-3)$$

P -45
S 12
F 15, -3

$$x + 15 = 0 \qquad x - 3 = 0$$

$$\cancel{x = -15} \qquad x = 3$$

length can't be -ve

∴ dimensions are
10m × 8m

$x+7$ $x+5$
$3+7$ $3+5$

Example 4: Determining Equations

Determine a quadratic equation which has roots of – 4 and $\frac{5}{3}$. solutions of a quadratic equation

$x = -4$ $x = \frac{5}{3}$

$x + 4 = 0$ $3x = 5$

$3x - 5 = 0$

$(x+4)(3x-5) = 0$ ← factored form

$3x^2 - 5x + 12x - 20 = 0$

$3x^2 + 7x - 20 = 0$ ← trinomial or general form

Bulawka's Bullets

☺ Make sure you know the difference between

Factor: $x^2 + x - 6$ ← expression and Solve $x^2 + x - 6 = 0$ – equation

Solution: $(x+3)(x-2)$ binomial factors *Solution:* $x = -3$ and $x = 2$ roots

☺ The RHS must be equal to 0 (or the zero product property does not apply).

☺ Make sure your solutions are logical for word problems ie. length can't be negative.

Exercise 3 Solving Quadratic Equations by Factoring

1.) Solve: (follow example 1)
a.) $x^2 + x - 20 = 0$
$x = -5, 4$

b.) $(x + 3)(2x - 5) = 0$
$x = -3, \frac{5}{2}$

2.) Solve: (follow example 2)
a.) $42 = x^2 - x$
$x = 7, -6$

b.) $3 = 6x^2 - 7x$
$x = -\frac{1}{3}, \frac{3}{2}$

3.) Solve: (distribute, then follow example 2)
$x(2x - 3) - 2(3 + 2x) = -4(x + 1)$
$x = -\frac{1}{2}, 2$

4.) Solve: (follow example 3)
a.) The product of two consecutive even numbers is 16 more than 8 times the smaller integer. Determine the integers. 8 and 10
b.) The width of the top of a notebook computer is 7 cm less than the length. The surface area of the top of the notebook is 690 cm^2. Determine the dimensions of the computer. 30 cm × 23 cm

5.) Determine an equation for: (follow example 4)
a.) a quadratic equation with roots of $\frac{2}{3}$ and 4.
$(3x - 2)(x - 4) = 0$
$3x^2 - 14x + 8 = 0$
b.) a quadratic equation which has factors of $(x - 1)$ and $(3x - 2)$.
$(x-1)(3x-2) = 0$ or $3x^2 - 5x + 2 = 0$

6.) Factor: (L2, ex. 2)
$(x - 2)^2 - 6(x - 2) - 16$

7.) Factor: (L1, ex. 4)
$4x^2 + 4x - 3$

Extra Practice: Pg. 190 #4-10, 14, 16, 18b, 20