# Lesson 2 Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad \cos^2\theta = 1 - \sin^2\theta \quad \sin^2\theta = 1 - \cos^2\theta$

$\tan^2\theta + 1 = \sec^2\theta \quad \tan^2\theta = \sec^2\theta - 1$

$\cot^2\theta + 1 = \csc^2\theta \quad \cot^2\theta = \csc^2\theta - 1$

**Ex. 1)** Prove the given identities for all permissible values of $\theta$.

a.) $\cot\theta + \tan\theta = \csc\theta \sec\theta$

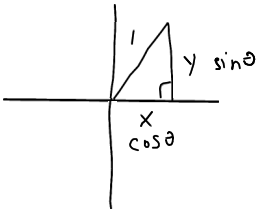
| Left-Hand Side | Right-Hand Side |
|---|---|
| $\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$ | $\csc\theta \sec\theta$ |
| LCD $\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$ (1) | |
| $\frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta}$ | |
| $\csc\theta \sec\theta$ | |
| LHS = RHS ✓ | |

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$




$$x^2 + y^2 = 1$$

$$\cos^2 x + \sin^2\theta = 1$$

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b.) $\cot^3\theta = \cot\theta\csc^2\theta - \cot\theta$

| Left-Hand Side | Right-Hand Side |
|---|---|
| $\cot^3\theta$ | $\cot\theta(\csc^2\theta - 1)$ |
| | $\cot\theta\cot^2\theta$ |
| | $\cot^3\theta$ |
| LHS = RHS ✓ | |

GCF $\cot\theta$

$\cot^2\theta + 1 = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

c.) $\frac{1+\cos^2\theta}{\sin^2\theta} = 2\csc^2\theta - 1$

| Left-Hand Side | Right-Hand Side |
|---|---|
| $\frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta}$ | $2\csc^2\theta - 1$ |
| $\csc^2\theta + \cot^2\theta$ | |
| $\csc^2\theta + \csc^2\theta - 1$ | |
| $2\csc^2\theta - 1$ | |
| LHS = RHS ✓ | |

Split into two parts

$\cot^2\theta = \csc^2\theta - 1$ sub

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d.) $\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

| Left-Hand Side | Right-Hand Side |
|---|---|
| $\frac{1-\cos\theta}{\sin\theta}$ | $\frac{\sin\theta}{1+\cos\theta}\cdot\frac{(1-\cos\theta)}{(1-\cos\theta)}$ |
| | $\frac{\sin\theta(1-\cos\theta)}{1-\cos\theta+\cos\theta-\cos^2\theta}$ |
| | $\frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}$ |
| | $\frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}$ |
| | $\frac{1-\cos\theta}{\sin\theta}$ |

$(1+\cos\theta)(1-\cos\theta)$ conjugates — same terms, different sign

$\sin^2\theta+\cos^2\theta=1$

$\sin^2\theta = 1-\cos^2\theta$

LHS = RHS ✓

Worksheet
# 10, 14, 17, 18, 19, 20, 21, 29, 30