

# Properties of Limits.notebook

## Properties of Limits

If  $L, M, c$  and  $k$  are real numbers and  $\lim_{x \rightarrow c} f(x) = L$   
and  $\lim_{x \rightarrow c} g(x) = M$  then:

① Sum Rule:  $\lim_{x \rightarrow c} (f(x) + g(x)) = L + M$

The limit of the sum of the two functions is the sum of their limits

② Difference Rule:  $\lim_{x \rightarrow c} (f(x) - g(x)) = L - M$

③ Product Rule:  $\lim_{x \rightarrow c} (f(x) \cdot g(x)) = L \cdot M$

④ Constant Multiple Rule:  $\lim_{x \rightarrow c} (k f(x)) = k \cdot L$  where  $k$  is some constant

⑤ Quotient Rule:  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{M}$ ,  $M \neq 0$  (or as long as  $\lim_{x \rightarrow c} g(x)$  exists)

⑥ Power Rule:  $\lim_{x \rightarrow c} (f(x))^n = L^n$  provided  $L^n$  is a real number

### Other Properties

$$\lim_{x \rightarrow a} (c) = c$$

$$\lim_{x \rightarrow a} x^n = a^n$$

$$\lim_{x \rightarrow c} (x) = c$$

$$\lim_{x \rightarrow a} \sqrt[n]{x} = \sqrt[n]{a}$$

$$\lim_{x \rightarrow 2} (\sqrt{3}x) = 2\sqrt{3}$$

ex. Use the properties of limits to find

$$\lim_{x \rightarrow c} (3x^3 + 2x - 9)$$

$$3 \lim_{x \rightarrow c} x^3 + 2 \lim_{x \rightarrow c} x - \lim_{x \rightarrow c} 9$$

$$3(\lim_{x \rightarrow c} x)^3 + 2 \lim_{x \rightarrow c} x - \lim_{x \rightarrow c} 9$$

$$3(c)^3 + 2c - 9$$

$$\therefore \lim_{x \rightarrow c} (3x^3 + 2x - 9) = 3c^3 + 2c - 9$$

note:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

← from graph on Desmos

Apply to  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

$$\lim_{x \rightarrow 0} \tan x \cdot \frac{1}{x}$$

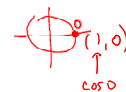
Using props

$$\lim_{x \rightarrow 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{1}{\cos x}$$

$$\frac{1}{\cos 0}$$

$$1 \cdot 1$$



pg 66 #9-11 Evaluate using props

pg 67 #24, 25, 40, 42, 44, 49, 51