Grade 10 Applied Pre-Calculus
Practice Exam

Factors and Products

1. Write 1022 as a product of its prime factors.

$1022 \div 2 = 511$ $\quad 73 \div 73 = 1$ $\quad \therefore 1022 = 2 \cdot 7 \cdot 73$

$511 \div 7 = 73$

2. Determine the greatest common factor of 64, and 120. Make sure you show your work.

$64 \div 2 = 32$ $\quad 4 \div 2 = 2$ $\quad 120 \div 2 = 60$ $\quad 15 \div 3 = 5$

$32 \div 2 = 16$ $\quad 2 \div 2 = 1$ $\quad 60 \div 2 = 30$ $\quad 5 \div 5 = 1$

$16 \div 2 = 8$ $\quad 30 \div 2 = 15$

$8 \div 2 = 4$

$\therefore GCF = 2 \cdot 2 \cdot 2 = 8$

3. Determine the least common multiple of 9, and 12. Make sure you show your work.

9, 18, 27, (36)

12, 24, (36)

LCM is 36

4. Two marching bands are to be arranged in rectangular arrays with the same number of columns. One band has 54 members, the other has 42 members. What is the greatest number of columns in the array?

$42 \div 2 = 21$ $\quad 54 \div 2 = 27$ $\quad 3 \div 3 = 1$ $\quad GCF = 2 \cdot 3 = 6$

$21 \div 3 = 7$ $\quad 27 \div 3 = 9$

$7 \div 7 = 1$ $\quad 9 \div 3 = 3$

5. What are the dimensions of the smallest square that could be tiled using an 18 cm by 24 cm tile?

18, 36, 54, (72)

24, 48, (72)

LCM = 72

6. Find the edge length of a cube with a volume of 1728. Show your work for full marks.

$1728 \div 2 = 864$ $\quad 108 \div 2 = 54$ $\quad 3 \div 3 = 1$ $\quad 2^2 \cdot 3$ $\quad 2^2 \cdot 3$ $\quad 2^2 \cdot 3$

$864 \div 2 = 432$ $\quad 54 \div 2 = 27$ $\quad 12 \cdot 12 \cdot 12$

$432 \div 2 = 216$ $\quad 27 \div 3 = 9$

$216 \div 2 = 108$ $\quad 9 \div 3 = 3$

$\sqrt[3]{1728} = 12$

7. Expand and simplify the following:

a) $(3x - 2)(x + 4)$

$3x^2 + 12x - 2x - 8$

$3x^2 + 10x - 8$

b) $(x - 1)^3$ $\quad (x-1)(x-1)$

$x^2 - 2x + 1 \,(x-1)$

$x^3 - x^2 - 2x^2 + 2x + x - 1 \Rightarrow x^3 - 3x^2 + 3x - 1$

c) $3x(2x - 4) - (x + 1)^2$

$6x^2 - 12x - (x+1)(x+1)$

$6x^2 - 12x - (x^2 + 2x + 1)$

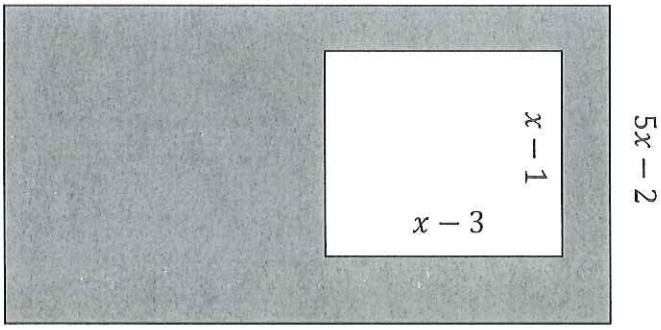
$6x^2 - 12x - x^2 - 2x - 1$

$5x^2 - 14x - 1$

8. Determine a simplified expression for the area of shaded region.

Lrg: $(5x-2)(3x+2)$
$15x^2+10x-6x-4$
$15x^2+4x-4$

Sml: $(x-3)(x-1)$
$x^2-x-3x+3$
x^2-4x+3



$15x^2+4x-4$
$-\ x^2-4x+3$
$14x^2+8x-7$

9. Factor the following:

a) $x^2-5x-14$ $(x-7)(x+2)$

b) $3x^2y^3-x^2y^2$ $x^2y^2(3y-1)$

c) $2x^2-11xy-6y^2$
P -12
S -11
F $-12, 1$
$-6 \quad 1$
$(2x+y)(x-6y)$

d) $12x^2+2x-4$
P -12
S 1
F $4, -3$
$2 \quad -1$
$2(6x^2+x-2)$
$2(2x-1)(3x+2)$

e) $9x^4-81y^4$ $(3x^2-9y^2)(3x^2+9y^2)$

f) x^4-3x^2-4 $(x^2-4)(x^2+1)$
$(x+2)(x-2)(x^2+1)$

Roots and Powers

10. Simplify and leave only positive exponents:

a) $2x^{-3}$ $\frac{2}{x^3}$

b) $4^{-\frac{1}{2}}$ $\frac{1}{4^{1/2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

c) $(2x^2y^3)(4x^{-2}y^4)^2$ $(2x^2y^3)(16x^{-4}y^8)$ $32x^{-2}y^{11} \Rightarrow \boxed{\frac{32y^{11}}{x^2}}$

d) $\frac{(3xy)^0(5x^2y^3)^2}{50xy}$ $\frac{25x^4y^6}{50xy} = \boxed{\frac{x^3y^5}{2}}$

e) $\left(\frac{27x^{-2}y^4}{3xy^{-1}}\right)^{-\frac{1}{2}}$ $\left(\frac{3xy^{-1}}{27x^{-2}y^4}\right)^{1/2} = \left(\frac{x^3y^{-5}}{9}\right)^{1/2} = \frac{x^{3/2}}{\sqrt{9}\,y^{5/2}} = \boxed{\frac{x^{3/2}}{3y^{5/2}}}$

11. Simplify the following radicals:

a) $\sqrt{208}$ $\sqrt{16}\cdot\sqrt{3} = 4\sqrt{3}$

b) $\sqrt[3]{108}$ $\sqrt[3]{27}\cdot\sqrt[3]{4} = 3\sqrt[3]{4}$

c) $\sqrt{24x^5y^4}$ $\sqrt{24} = \sqrt{4}\cdot\sqrt{6} = 2\sqrt{6}$ $\boxed{x\cdot x}$ $\boxed{x\cdot x}\cdot x$ $\boxed{y\cdot y}$ $\boxed{y\cdot y}$ $2x^2y^2\sqrt{6x}$

12. Change the following mixed radicals to an entire radical.

a) $4\sqrt{5}$ $\sqrt{16}\cdot\sqrt{5} = \sqrt{80}$

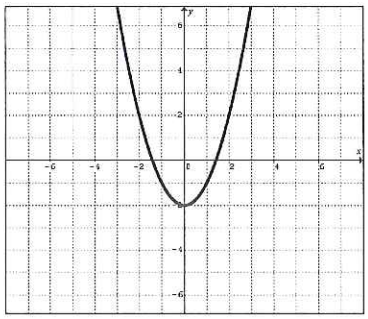
b) $2\sqrt[4]{3}$ $\sqrt[4]{24}\cdot\sqrt[4]{3} = \sqrt[4]{16}\cdot\sqrt[4]{3} = \sqrt[4]{48}$

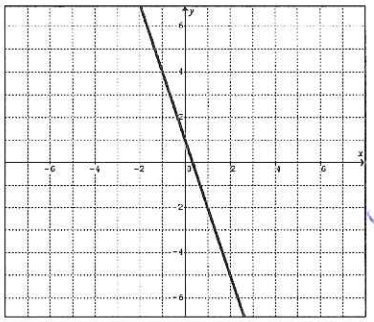
Relations and Functions

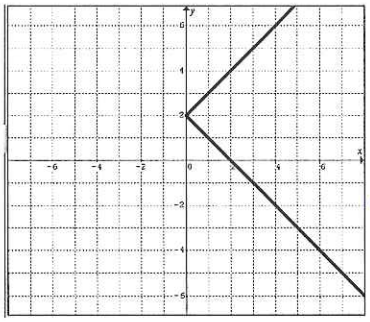
13. For each of the following below, determine:

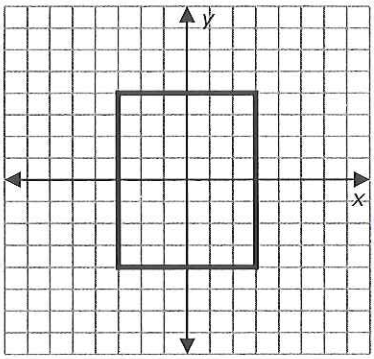
- domain
- range
- *y*-intercept
- state whether it is a function or not, explain your reasoning

a) $\{(3,4),(-2,-1),(4,-6),(-2,2)\}$ D: $x = 3, -2, 4, -2$ y-int = none
R: $y = 4, -1, -6, 2$ Fnc = no

b) 
D: $x \in \mathbb{R}$
R: $y \geq -2$
yint: -2
Fnc: yes

c) 
D: $x \in \mathbb{R}$
R: $y \in \mathbb{R}$
yint: 1
Fnc: yes

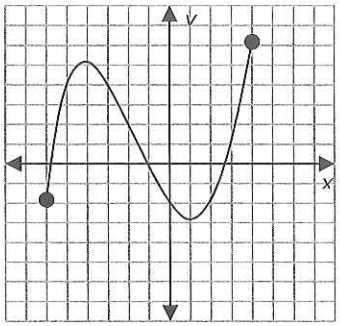
d) 
D: $x \geq 0$
R: $y \in \mathbb{R}$
yint: 2
Fnc: no

e) 

D: $-3 \leq x \leq 3$
R: $-4 \leq y \leq 4$
yint: ± 4
Fnc: no

14. Given the following three functions:

$f(x) = 3x - 5$ $g(x) = x^2 - 4$ $h(x) =$



Find:

a) $f(3)$ $3(3)-5 = 4$

b) $g(0)$ $0^2-4 = -4$

c) $h(-3)$ $= 4$

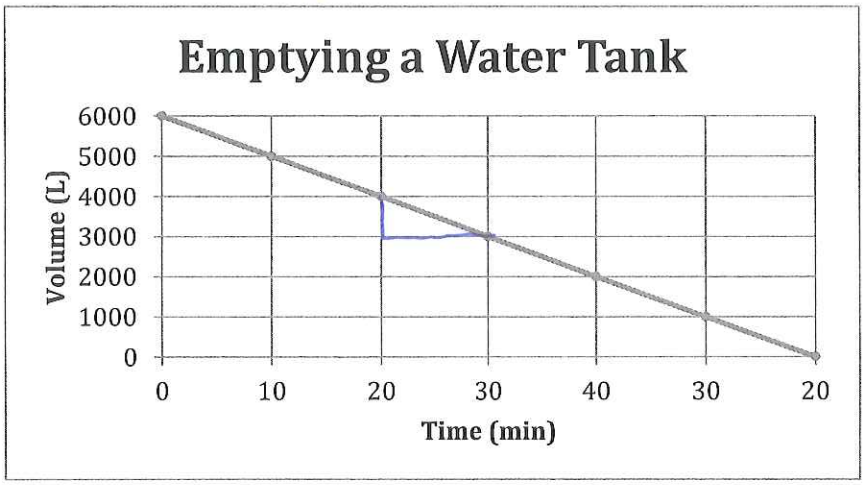
d) $f(a)$ $3a-5$

e) $f(-1) + g(2)$

$3(-1)-5 \quad 2^2-4$

$-8 + 0 = -8$

15.



a) Determine the independent and dependent variables.

I = Time D = Volume

b) At 30 minutes, what is the volume of water remaining in the tank?

3000L

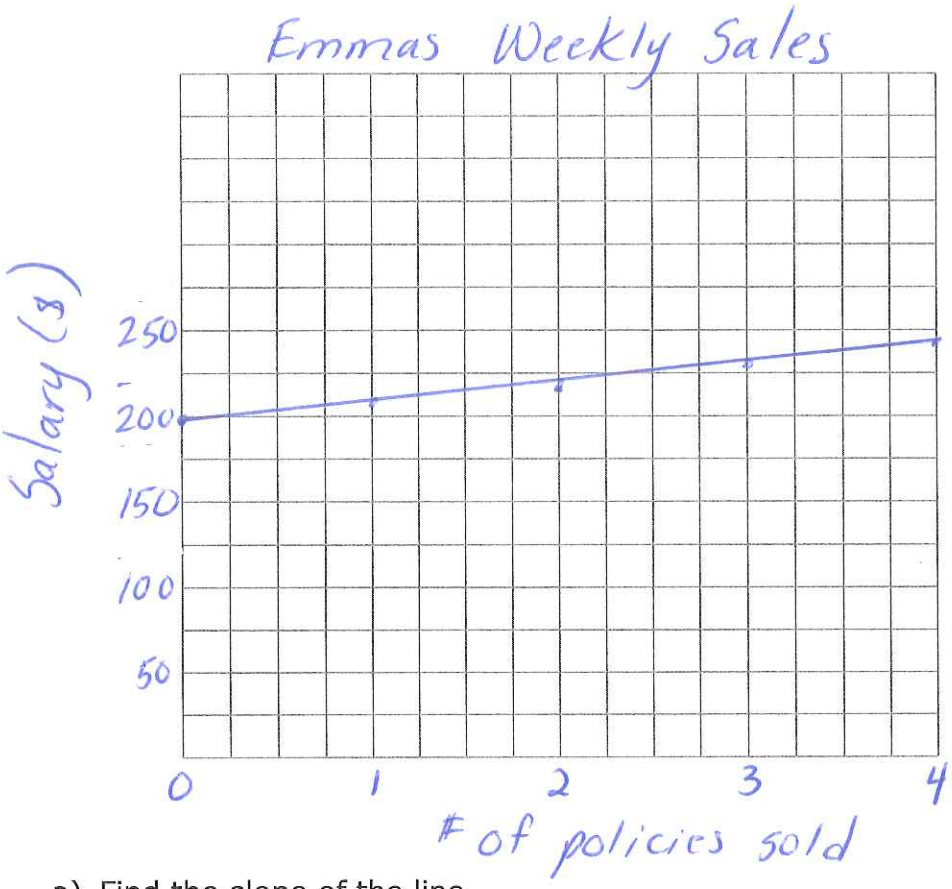
c) What is the rate of the change of the graph? What does it represent?

$m = \frac{-1000}{10} = -100$

The volume of the tank is decreasing at a rate of 100L/min

16. Emma is an insurance sales person. She earns a base salary of $200 per week, plus $10 for every policy she sells. Generate some data, and graph the relation. Remember to include all titles, and labeling.

# of policies sold	0	1	2	3	4
Salary	200	210	220	230	240



a) Find the slope of the line.

$m = 10$

b) What does the slope of the line represent?

$10 per policy sold

c) Write an equation to represent the linear function.

$y = 10x + 200$

d) If Emma sells 10 policies, how much is her gross pay? Use the formula to answer the question.

$y = 10(10) + 200$
$y = 100 + 200$
$y = 300$

gross pay is $300

Linear Functions

17. Calculate the distance between (-5, 4) and (1, -9). Leave answer as a simplified radical. $\sqrt{(-5-1)^2+(4-(-9))^2}$ $\sqrt{(-6)^2+(13)^2}$ $\sqrt{36+169}$ $\sqrt{182}$

18. Find the midpoint between (1, 3) and (-5, 10). $M=\left(\frac{1+(-5)}{2}, \frac{3+10}{2}\right) \Rightarrow \left(\frac{-4}{2}, \frac{13}{2}\right) \Rightarrow (-2, 13/2)$

19. If the midpoint of AB is (5, -4) and A is (0, -6), find the coordinates of point B. $(5,-4)=\left(\frac{0+x_2}{2}, \frac{-6+y_2}{2}\right)$ $5=\frac{0+x_2}{2}$ $10=x_2$ $-4=\frac{-6+y_2}{2}$ $-8=-6+y_2$ $-2=y_2$ $\therefore B(10,-2)$

20. Find the slope of the line that goes through the points (10, 12) and (-6, 9). $m=\frac{9-12}{-6-10}=\frac{-3}{-16}=\frac{3}{16}$

21. Re-write the following linear equations in slope intercept form:

a) $y-3=\frac{2}{3}(x+4)$ $\overset{\times 3}{}\ \overset{\times 3}{}\ \overset{\times 3}{}$ $3y-9=2x+8$ $3y=2x+17$ $y=\frac{2}{3}x+\frac{17}{3}$

b) $3x-4y+8=0$ $3x+8=4y$ $\frac{3}{4}x+2=y$

c) $3x-y=\frac{4}{5}$ $3x-\frac{4}{5}=y$

22. Find the equation of the line that passes through (5, 8) and has a slope of $\frac{3}{2}$. ***Answer in general form.*** $y-8=\frac{3}{2}(x-5)$ $2y-16=3x-15$ $0=3x-2y+1$

23. Find the equation of the line that passes through the points (-1, 6) and (5, -4). ***Answer in slope intercept form.*** $m=\frac{-4-6}{5-(-1)}=\frac{-10}{6}=\frac{-5}{3}$ $y-6=\frac{-5}{3}(x+1)$ $3y-18=-5x-5$ $3y=-5x+13$ $y=\frac{-5}{3}x+\frac{13}{3}$

24. Find the equation of the line that passes through the point (3, 7) and has a *y*-intercept of -2. ***Answer in general form.*** $(0,-2)$ $m=\frac{-2-7}{0-3}=\frac{-9}{-3}=3$ $y=3x-2$ $0=3x-y-2$

25. Find the equation of the line that is perpendicular to the line $4x - y + 10 = 0$ and has an x-intercept of 7. ***Answer in slope intercept form.***

$4x+10=y$, $m=4$, $\perp m = -\frac{1}{4}$

$(7,0)$ $y-0=-\frac{1}{4}(x-7)$ $\boxed{y=-\frac{1}{4}x+\frac{7}{4}}$

26. Find the equation of the line that is parallel to the line $y - 3 = \frac{1}{2}(x + 2)$ and passes through the point (4, -6). ***Answer in general form.***

$\parallel m = \frac{1}{2}$

$y+6=\frac{1}{2}(x-4)$ $\boxed{0=x-2y-16}$

$2y+12=x-4$

27. Graph the linear function $y = -\frac{4}{5}x + 4$ using the intercepts method of graphing.

let $x=0$

$y=-\frac{4}{5}(0)+4$

$y=4$

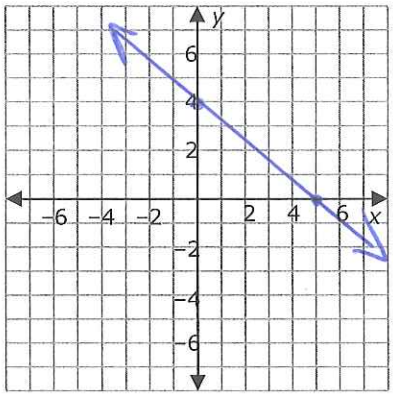
let $y=0$

$0=-\frac{4}{5}x+4$

$0=-4x+20$

$4x=20$

$x=5$



28. Graph the linear function $3x - 4y + 16 = 0$ using the slope intercept method of graphing.

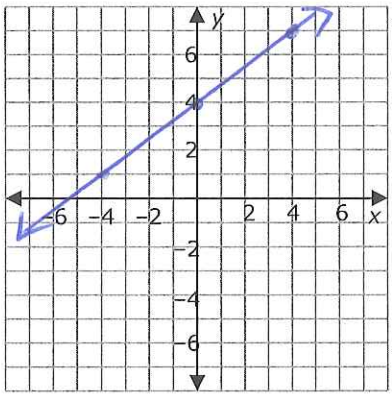
$3x-4y+16=0$

$3x+16=4y$

$\frac{3}{4}x+4=y$

$m=\frac{3}{4}$ up 3 r+4

$y_{int}=4$



29. Determine the x and y-intercepts for $5x - 3y + 12 = 0$.

x_{int} let $y=0$

$5x-3(0)+12=0$

$5x=-12$

$\boxed{x=-\frac{12}{5}}$

$y_{int} \Rightarrow$ let $x=0$

$5(0)-3y+12=0$

$12=3y$

$\boxed{4=y}$

Systems of Linear Equations

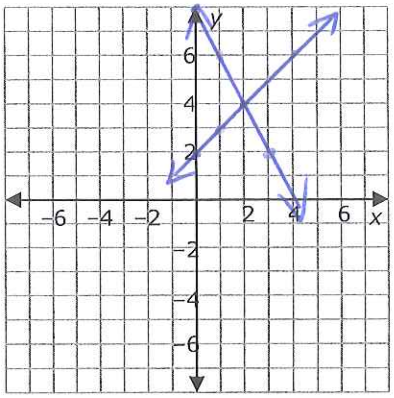
30. Solve the system of linear equations by graphing.

$x - y = -2$
$4x + 2y = 16$

$x + 2 = y$

$2y = -4x + 16$
$y = -2x + 8$

$\therefore (2, 4)$



31. Solve the system of linear equations using substitution.

$3x + y = 3$
$2x + 3y = -5$

$y = -3x + 3$

$2x + 3(-3x + 3) = -5$
$2x - 9x + 9 = -5$
$-7x = -14$
$x = 2$

$y = -3(2) + 3$
$y = -6 + 3$
$y = -3$

$\therefore (2, -3)$

32. Solve the system of linear equations using elimination.

$5x + 2y = 5$ x2
$3x - 4y = -23$

$10x + 4y = 10$
$3x - 4y = -23$
$13x = -13$
$x = -1$

$5(-1) + 2y = 5$
$2y = 10$
$y = 5$

$\therefore (-1, 5)$

33. A play-off football game drew 36 500 fans. Depending on seat location, the ticket prices were \$35 and \$20. The total revenue from the ticket sales was \$940 000. How many \$35 tickets and how many \$20 tickets were sold?

x - \$35 Tickets
y - \$20 Tickets
$x + y = 36500$
$35x + 20y = 940,000$

$y = -x + 36500$

$35x + 20(-x + 36500) = 940,000$
$35x - 20x + 730,000 = 940,000$
$15x = 210,000$
$x = 14,000$

$y = 36500 - 14,000$
$y = 22,500$

$\therefore$ 14,000 - \$35 Tickets
22,500 - \$20 Tickets

34. Three footballs and one soccer ball cost \$155. Two footballs and three soccer balls cost \$220. Determine the cost of one football and the cost of one soccer ball.

x = cost of footballs
y = cost of soccer balls

$3x + y = 155$ x3
$2x + 3y = 220$

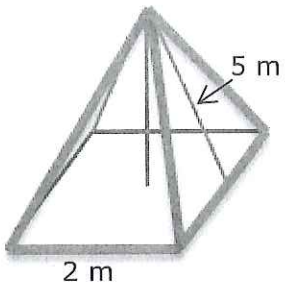
$3(35) + y = 155$
$y = 155 - 105$
$y = 50$

$9x + 3y = 465$
$-2x + 3y = 220$
$7x = 245$
$x = 35$

$\therefore$ \$35 for footballs
\$20 for soccer balls

Measurement

35. Calculate the surface area of this pyramid:



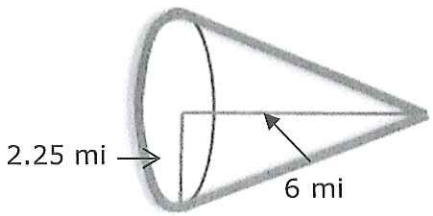
$SA = 2bs + b^2$

$= 2(2)(5) = 2^2$

$= 20 + 4$

$= \boxed{24 m^2}$

36. Calculate the surface area of this cone:



$6^2 + 2.25^2 = s^2$

$36 + 5.06 = s^2$

$\sqrt{41.0625} = \sqrt{s^2}$

$6.4 = s$

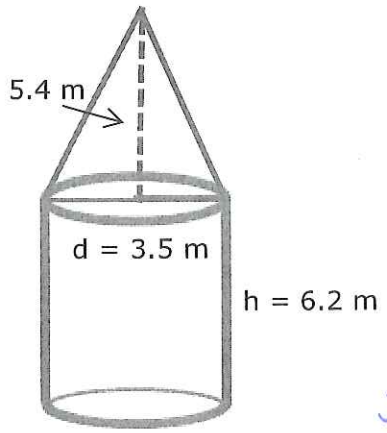
$SA = \pi rs + \pi r^2$

$= \pi(2.25)(6.4) + \pi(2.25)^2$

$= 45.3 + 15.90$

$= \boxed{61.20 mi^2}$

37. Determine the surface area for the following composite figure.



$5.4^2 + 1.75^2 = s^2$

$29.16 + 3.06 = s^2$

$\sqrt{32.2225} = \sqrt{s^2}$

$5.68 = s$

Cone - lateral

$SA = \pi rs$

$= \pi(1.75)(5.68)$

$= 31.21$

Cylinder

$SA = 2\pi rh + \pi r^2$

$= 2\pi(1.75)(6.2) + \pi(1.75)^2$

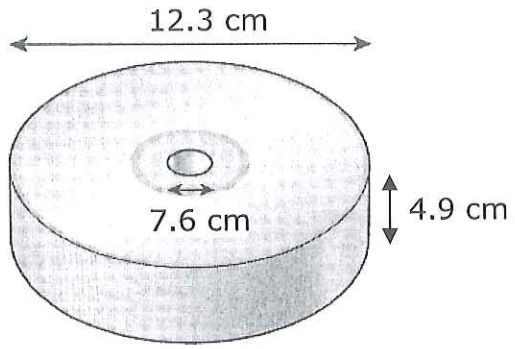
$= 68.17 + 9.62$

$= 77.79 m^2$

Total $= 31.21 + 77.79$

$= 109.00 m^2$

38. A roll of duct tape has the following dimensions. There is a cylindrical hole in the centre of the shape. Calculate the volume of the tape.



Lrg Cylinder
$V = \pi(6.15)^2(4.9)$
$= 582.23$

Sml Cylinder
$V = \pi(3.8)^2(4.9)$
$= 222.29$

Total $= 582.23 - 222.29$
$= 359.94\ cm^3$

39. A cube has a volume of 27 cm^3. What is its surface area?

$V = s^3$
$\sqrt[3]{27} = \sqrt[3]{s^3}$
$3 = s$

$SA = 6s^2$
$= 6(3)^2$
$= \boxed{54\ cm^2}$

40. If the radius of a cylinder is 3 cm, and the total surface area is 42π cm^2, what is the height?

$SA = 2\pi rh + 2\pi r^2$
$42\pi = 2\pi(3)h + 2\pi(3)^2$
$42\pi - 18\pi = 6\pi h + 18\pi - 18\pi$
$\frac{24\pi}{6\pi} = \frac{6\pi h}{6\pi}$

$\boxed{4cm = h}$

41 Convert the following measurements:

a) 48 inches to 4 ft
÷12

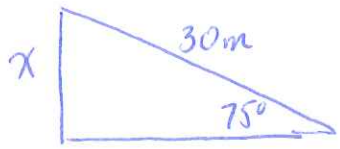
b) 72 ft to 21.95 m
×0.3048

c) 100 cm to 1 m
÷100

d) 74 inches to 187.96 cm
×2.54

Trigonometry

42. A 30-m long line is used to hold a helium weather balloon. Due to a breeze, the line makes a 75° angle with the ground. Find the height of the balloon.

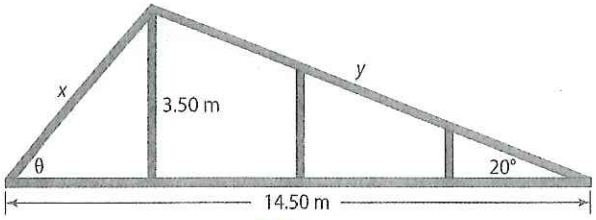


$\sin 75° = \frac{x}{30}$

$x = 30(\sin 75°)$

$x = 28.98\text{m}$

43. Michael is building a cabin at Cold Lake, AB. He has drawn a diagram to design his roof truss. Determine the values of *x*, *y*, and θ.



$\tan\theta = \frac{3.5}{4.88}$

$\theta = \tan^{-1}\left(\frac{3.5}{4.88}\right)$

$\theta = 35.6°$

$y = \sin 20° = \frac{3.5}{y}$

$y = \frac{3.5}{\sin 20°}$

$y = 10.23\text{m}$

$\tan 20° = \frac{3.5}{x}$

$x = \frac{3.5}{\tan 20°}$

$x = 9.62$

$14.5 - 9.62 = 4.88$

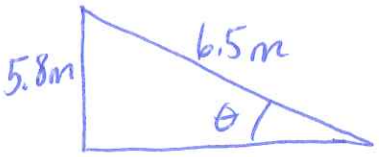
x

$\sin 35.6° = \frac{3.5}{x}$

$x = \frac{3.5}{\sin 35.6°}$

$x = 6.01\text{m}$

44. A cat is on a tree branch 5.8 m above the ground. If a ladder 6.5 metres long is placed on the branch, what angle does the ladder make with the ground?

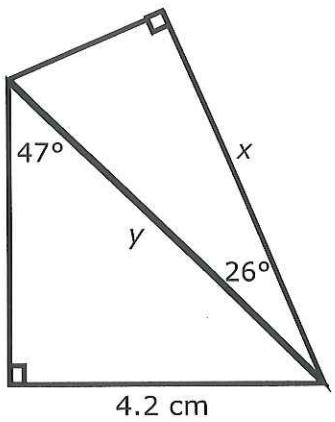


$\sin\theta = \frac{5.8}{6.5}$

$\theta = \sin^{-1}\left(\frac{5.8}{6.5}\right)$

$\theta = 63.16°$

45. Calculate the length of side x.



$\sin 47° = \frac{4.2}{y}$

$y = \frac{4.2}{\sin 47°}$

$y = 5.74\text{cm}$

$\cos 26° = \frac{x}{5.74}$

$x = 5.74(\cos 26°)$

$x = 5.16\text{cm}$