

Polynomial Functions

January 2014

Question 26

2 marks

One of the factors of $P(x) = x^3 - kx^2 - 7x + 10$ is $(x - 2)$.

Find the value of k .

Solution**Method 1**

$$x = 2$$

½ mark for $x = 2$

$$0 = (2)^3 - k(2)^2 - 7(2) + 10$$

1 mark for remainder theorem

$$0 = 8 - 4k - 14 + 10$$

$$0 = 4 - 4k$$

$$4k = 4$$

$$k = 1$$

½ mark for solving for k

2 marks

Question 35

1 mark

When $P(x)$ is divided by $x - 3$, it has a quotient of $2x^2 + x - 6$ and a remainder of 4.

Determine $P(x)$.

Solution

$$P(x) = (x - 3)(2x^2 + x - 6) + 4$$

1 mark for polynomial $P(x)$

or

1 mark

$$P(x) = 2x^3 - 5x^2 - 9x + 22$$

Question 40

4 marks

Sketch the graph of $y = x^3 + x^2 - 5x + 3$ given that one of the x -intercepts is 1.

Identify the x -intercepts and y -intercept.

Solution

$$x = 1$$

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -5 & 3 \\ & & 1 & 2 & -3 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

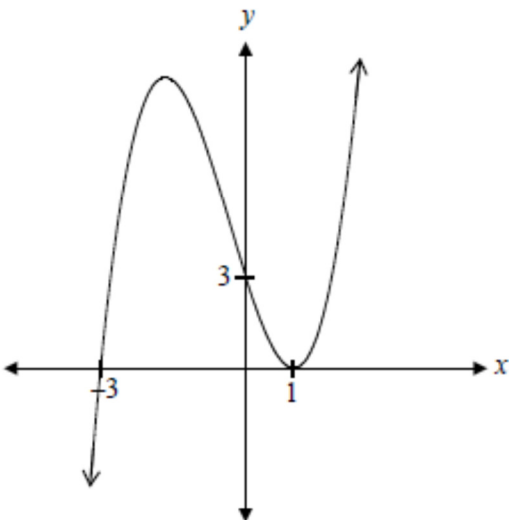
1 mark for synthetic division

$$y = (x - 1)(x^2 + 2x - 3)$$

$$y = (x - 1)(x + 3)(x - 1)$$

$$y = (x + 3)(x - 1)^2$$

1 mark for identifying the factors



2 marks for graph ($\frac{1}{2}$ mark for x -intercepts, $\frac{1}{2}$ mark for multiplicity, $\frac{1}{2}$ mark for y -intercept, $\frac{1}{2}$ mark for end behaviour)

4 marks

June 2013**Question 44****2 marks**

Is $(x - 3)$ a factor of $x^4 - x^3 - 3x^2 + x - 1$?

Justify your answer.

Solution**Method 1**

$$x = 3$$

$$\begin{aligned} \therefore (3)^4 - (3)^3 - 3(3)^2 + (3) - 1 &= 81 - 27 - 27 + 3 - 1 \\ &= 29 \end{aligned}$$

The remainder does not equal zero,
therefore $(x - 3)$ is not a factor.

½ mark for $x = 3$

1 mark for remainder theorem

½ mark for explanation

2 marks

Method 2

$$\begin{array}{r|rrrrr} 3 & 1 & -1 & -3 & 1 & -1 \\ & \downarrow & 3 & 6 & 9 & 30 \\ \hline & 1 & 2 & 3 & 10 & 29 \end{array}$$

The remainder does not equal zero,
therefore $(x - 3)$ is not a factor.

½ mark for $x = 3$

1 mark for synthetic division

½ mark for explanation

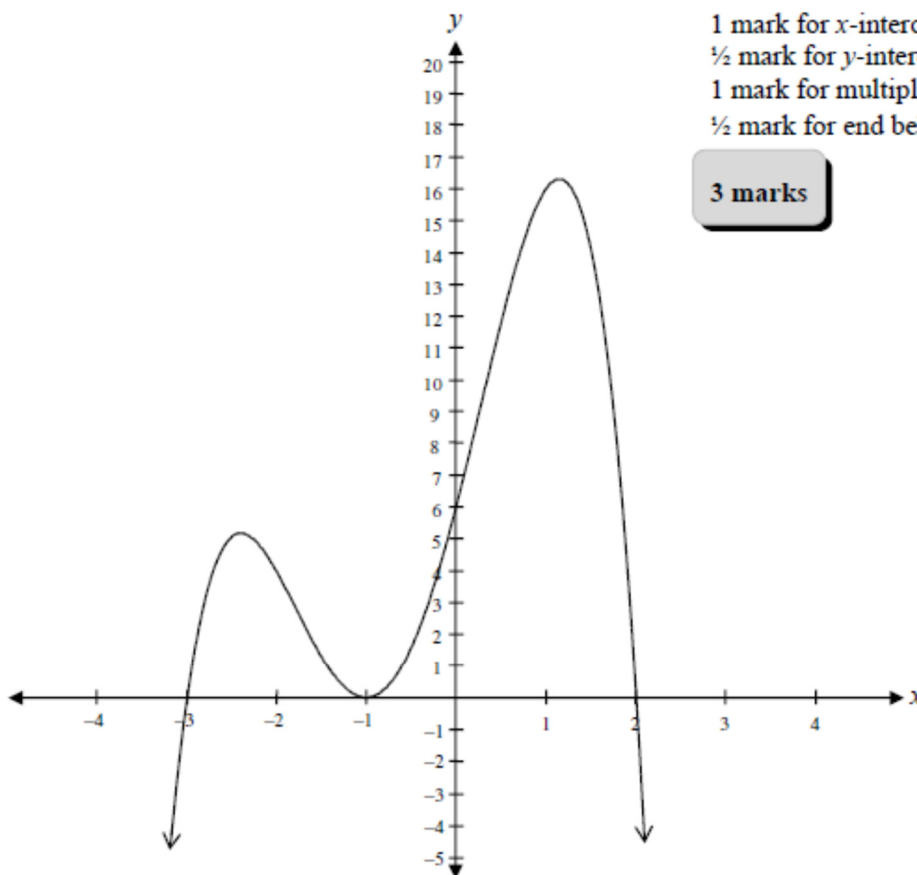
2 marks

Question 29**3 marks**

Sketch the graph of:

$$f(x) = (2 - x)(x + 3)(x + 1)^2$$

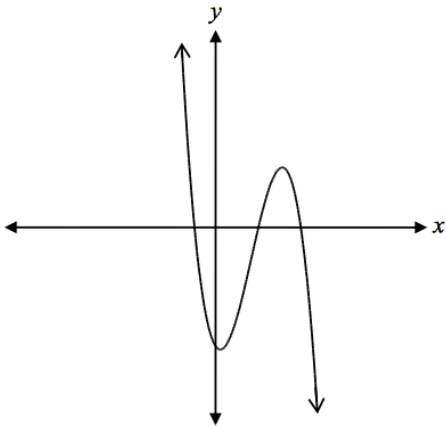
Label the x-intercepts and y-intercept.

Solutionx-intercepts: -3 , -1 , and 2 y-intercept: 6 

Question 31

1 mark

The graph below represents the equation $y = ax^3 + 6x^2 + 5x - 10$.



What must be true about the value of a ? Explain your reasoning.

Solution

a is any negative number.

½ mark

Explanation with reference to end behaviour.

½ mark for explanation

1 mark

or

a cannot be zero.

½ mark

The graph is of a cubic function,
not a quadratic function.

½ mark for explanation

1 mark