## Pre-Calculus 12 Enriched <br> Polynomial Functions \& Equations Extension

1.) A quadratic function is of the form $f(x)=x^{2}+b x+c$. The roots of the equation $f(x)=0$ are 1 and $k$. If $f(2)=5$, determine the value of $k$.
2.) The parabola $y=x^{2}-2 x+4$ is translated $p$ units to the right and $q$ units down. The $x$-intercepts of the resulting parabola are 3 and 5. Determine the values of $p$ and $q$.

3a.) For the quadratic equation $a x^{2}+b x+c=0$, where $a \neq 0$, show that the sum of the roots is $-\frac{b}{a}$ and the product of the roots is $\frac{c}{a}$.
b.) The roots of $x^{2}+c x+d=0$ are $a$ and $b$. The roots of $x^{2}+a x+b=0$ are c and d. If $a, b, c$, and $d$ are all non-zero, determine the value of $a+b+c+d$.
4.) Prove that the line with equation $y=2 x-1$ does not intersect the curve with equation $y=x^{4}+3 x^{2}+2 x$.
5.) Identify the degree of the polynomial and find the value of $f(x)$ that satisfies the following:
a.) $f(x)-f(x-1)=4$ with $f(0)=4$
b.) $f(x)-2 f(x-1)+f(x-2)=6$ with $f(1)=6, f(0)=1$
6.) Suppose that m and n are real numbers for which the three (not necessarily distinct) roots of $x^{3}-m x^{2}+n x-1=0$ are $m, n$, and 1 . Determine the value of $m+n+1$.
7.) Determine all values of $x$ that solve the equation.

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\left(x^{2}-3 x+1\right)^{2}-3\left(x^{2}-3 x+1\right)+1=x
$$

8.) The remainder when $f(x)=x^{5}-2 x^{4}+a x^{3}-x^{2}+b x-2$ is divided by $x+1$ is -7 . When $f(x)$ is divided by $x-2$, the remainder is 32 . Determine the remainder when $f(x)$ is divided by $x-1$.

