Pre-Calculus 11 Piece-wise Notation

Piece-wise Notation is used to describe a function that has different definitions for different subsets of the domain. The absolute value of a number is often defined using piece-wise notation.

x	-2	-1	0	1	2
y	-2	-1	0	1	2
y = |x|	2	1	0	1	2

y = -x

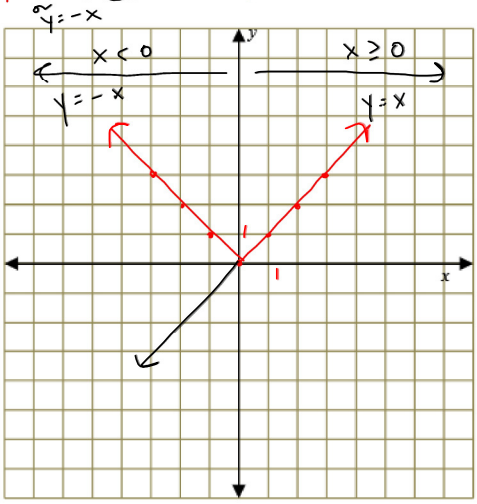
Graph $y = |x|$

Notice how the graph of $y = |x|$ is really the graph of $y = x$ *and* $y = -x$, joined together at the point (0, 0), the critical value.

We could write this in piece-wise notation as:

$$y = \begin{cases} x, \text{if } x \geq 0 \\ -x, \text{if } x < 0 \end{cases}$$

x-int



To write an absolute value function in piece-wise notation, we need to identify when the expression in the absolute value symbols is positive or zero, and negative.

Write the function $y = |x + 2|$ in piece-wise notation.

The critical value or x-intercept is –2.

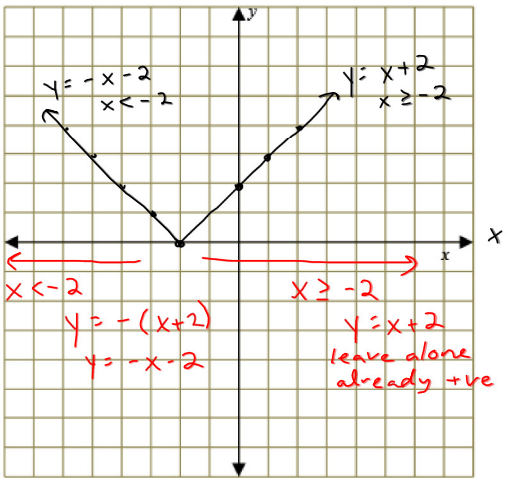
When $x \geq -2$, the $|x + 2| = x + 2$ for $x \geq -2$

When $x < -2$, the $|x + 2| = -(x + 2)$
or $-x - 2$ for $x < -2$.

Using Piece-wise Notation:

$$y = \begin{cases} x + 2, & if\ x \geq -2 \\ -x - 2, & if\ x < -2 \end{cases}$$

x-int



Examples: Write each function in piece-wise notation.

1. $y = |2x - 1|$

① Determine x-intercept

$y = 2x - 1$
$0 = 2x - 1$
$1 = 2x$
$\frac{1}{2} = x$

↑ critical value, turning point

separates domain subsets

$$y = \begin{cases} 2x-1 & x \geq \frac{1}{2} \\ -2x+1 & x < \frac{1}{2} \end{cases}$$

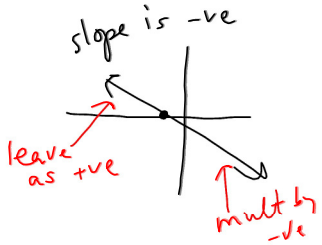
x-int

left of $x = \frac{1}{2}$ we change the eqn to $y = -(2x-1)$ or $y = -2x+1$

2. ~~$y = |4x - 3|$~~

$y = |-4x - 3|$

$y = -4x - 3$
$0 = -4x - 3$
$3 = -4x$
$-\frac{3}{4} = x$



$$y = \begin{cases} -4x-3 & x \leq -\frac{3}{4} \\ 4x+3 & x > -\frac{3}{4} \end{cases}$$

3. $y = |-x^2 + 2x + 3|$

$y = -x^2 + 2x + 3$

$0 = -x^2 + 2x + 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

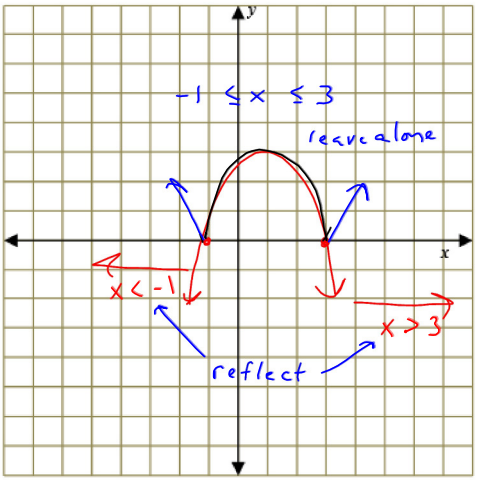
$x = 3 \quad x = -1$

x-ints $-1, 3$

$$y = \begin{cases} -x^2+2x+3 & -1 \le x \le 3 \\ x^2-2x-3 & x < -1 \cup x > 3 \end{cases}$$

union (or)

x-ints

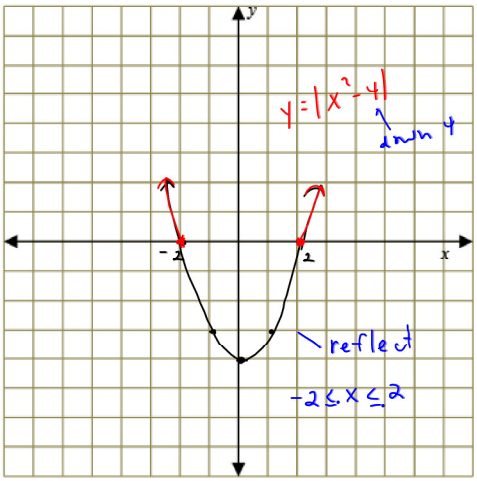


4. $y = |x^2 - 4|$

x-ints $-2, 2$

$$y = \begin{cases} x^2-4 & x \le -2 \cup x \ge 2 \\ -x^2+4 & -2 < x < 2 \end{cases}$$

$y = -(x^2-4)$



Assignment: Pg. 623; #9, ~~11~~ 15a