

Pre-Calculus 12 Permutations with Identical Objects

The number of permutations of n objects with r identical objects is:

$$\frac{n!}{r!}$$

Ex. 1) Determine the number of permutations of the word BOOK.

BOOK
 ↳ switching O's
 doesn't change
 the word

$$\frac{4!}{2!} = 12 \text{ ways}$$

↖ O's that won't change the arrangement
 if they switch positions

Given a set of n objects with:

- n_1 of one kind
- n_2 of a second kind
- n_3 of a third kind etc

The number of **distinguishable** permutations is:

$$\frac{n!}{n_1! \cdot n_2! \cdot \dots \cdot n_k!}, \text{ where } n_1 + n_2 + n_3 + \dots + n_k = n$$

Ex. 2) Determine the number of permutations of the word

a.) HONOLULU

need
brackets
on
calc

$$\frac{8!}{(2! \cdot 2! \cdot 2!)} = 5040 \text{ ways}$$

2 Os, 2 Ls, 2 Us

b.) MISSISSIPPI

$$\frac{11!}{(4! \cdot 4! \cdot 2!)} = 34650 \text{ ways}$$

* check on
calc to make
sure you get
the same
answer

Permutations with Identical Objects.notebook

Ex. 3) Seven boxes of cereal on a shelf are 5 Shreddies, 1 Fruit Loops and 1 box of Captain Crunch. How many ways can the boxes be arranged?

$$\frac{7!}{5!}$$

Use factorial expansion

$$\frac{7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!}}$$

simplify

42 ways

Ex. 4) A kabob recipe calls for 3 mushrooms, 4 shrimp, 2 cherry tomatoes and 5 slices of red pepper. How many ways can you arrange the items on a skewer?

need brackets on calc → $\frac{14!}{(3! \cdot 4! \cdot 2! \cdot 5!)} = 2522520 \text{ ways}$

Restrictions

Ex. 5) How many whole numbers are less than 300 (no repetition)?

case 1: 1 digit
10

case 2: 2 digits

$$\frac{9}{\text{not 0}} \cdot \frac{9}{\text{not 0}} = 81$$

case 3: 3 digits

$$\frac{2}{1 \text{ or } 2} \cdot \frac{9}{\text{not 0}} \cdot \frac{8}{\text{not 0}} = 144$$

$$\begin{aligned} \text{Total} &= 10 + 81 + 144 \\ &= 235 \text{ ways} \end{aligned}$$

Permutations with Identical Objects.notebook

Ex. 6) Using all of the letters of the word PARALLELOGRAM,

a) how many arrangements can be made using all of the letters?

$$\frac{13!}{(3! \cdot 2! \cdot 3!)} = 86\,486\,400$$

b) how many of these arrangements have all the L's together?

group of L's + 10 others \rightarrow $\frac{11!}{(3! \cdot 2!)} = 3\,326\,400$ arrangements

2 R's, 3 A's

c) how many of these arrangements have all of the A's together?

same as
b) since
3 A's $\quad 3\,326\,400$ arrangements

d) how many of these arrangements have all the R's together?

2 R's so 1 group of R's and 11 other letters } 12 groups $\frac{12!}{(3! \cdot 3!)} = 13\,305\,600$ arrangements

e) how many of these arrangements have all the L's, all the A's, and all the R's together?

1 group of L's
1 " of A's
1 " of R's
5 others } 8 groups $8! = 40\,320$ arrangements

worksheet
#1-4, 7, 9

Assignment: Pg. 712; #3a, c, 4b, d, 5a, c, 7, 8, 9, 12, MC# 1-3