Pre-Calculus 12 Permutations of Different Objects

To permute a set of objects means to arrange them.

A permutation is an arrangement of objects in a definite order (order is important)

n! represents the number of permutations of *n* different/distinct objects

Ex. 1) Given the word PHONE, how many 5-letter permutations of these letters can be created.

↑ 5 distinct letters

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

or

$n!$

$5! = 120$ perms

$_nP_r$

$_5P_5$

The number of permutations of n distinct objects taken r at a time is:

$$nPr = \frac{n!}{(n-r)!}, n \geq r$$

on formula sheet

Ex. 2) Evaluate $_9P_4$ "Permutation of 9 things taking only 4 at a time"

$_9P_4 = \frac{9!}{(9-4)!}$

$= \frac{9!}{5!}$

$= 3024$

or on Calc

9 nPr 4 = 3024

↑ could be in MATH options if not a key

Ex. 3) Given the word WINTER how many permutations of three letters are possible.

$_6P_3 = 120$

or $\frac{6!}{3!}$

$\frac{6 \cdot 5 \cdot 4 \cdot 3!}{3!}$

or $6 \cdot 5 \cdot 4$

Ex. 4) Eight students are competing in a 200 m race. How many ways can students finish 1st, 2nd and 3rd.

$_8P_3$ = 336 ways

using formula from formula sheet

$\frac{8!}{(8-3)!}$ $\frac{8!}{5!}$ $\frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!}}$

Ex. 5) Erin, Jill, Chris, Krista, and Larissa are off to Silvercity. Just prior to heading out, Krista and Larissa have a falling out over a math problem! In how many ways can the girls sit in a row at the movies if Krista and Larissa refuse to sit next to each other?

5!

all ways — ways sit together

5! — 4! 2!

72 ways

ways together

E, J, C, KL

4 grps 4!

switch places 2!

arrangement of groups

Ex. 6) In how many ways can four girls and three boys be arranged in a row in each situation?

a) A boy must be at each end of the row.

$\frac{3}{1st} \cdot 5! \cdot \frac{2}{last}$ = 720 ways

4 girls: other boy

b) The boys must be together.

5! 3! = 720 ways

bbb g g g g

5 grps 5!

3!

c) The girls must be together.

4! 4! = 576 ways

d) The ends of the row must be either both boys or both girls.

case 1: both boys (part a)

720

case 2: both girls

$\frac{4}{1st} \cdot 5! \cdot \frac{3}{last}$ = 1440

3 boys ? other 2 girls

Total ways = 720 + 1440

= 2160 ways

Ex. 7) Solve for n: ${}_nP_2=56$

$$nPr = \frac{n!}{(n-r)!}$$

$$56 = \frac{n!}{(n-2)!}$$

$$56 = \frac{n(n-1)(n-2)!}{(n-2)!}$$

$$0 = n^2 - n - 56$$

$$0 = (n-8)(n+7)$$

$n = 8$ $\quad n = -7$ rej

Ex. 8) Solve for r: ${}_5P_r = 20$

$$nPr = \frac{n!}{(n-r)!}$$

$$20 = \frac{5!}{(5-r)!}$$

$$20(5-r)! = 120$$

$$(5-r)! = 6$$

$$3! = 6$$

$\therefore (5-r) = 3$

$5 - 3 = r$

$2 = r$

$\therefore r = 2$

Ex. 9) A book collector has 5 Italian, 3 Spanish and 3 Greek books. In how many ways can he arrange these 11 books if the books of the same language must be kept together?

3 grps

5 Ital $\quad$ 3 Span $\quad$ 3 Greek

$3! \cdot 5! \cdot 3! \cdot 3!$

25 920 ways

2, 4, 5, 6, 7, 9, 11, 14,

pg 524 # 2 a, c, 6, 7 a, c, d, 10, 11, 12

Assignment: ~~Pg 702, #1a, b, 2, 6a, b, 7, 10a, c, d, 11, 12, 13, 14~~