# **Permutations/Combinations/Binomial Theorem**

## January 2014

## Question 4 (calculator)

2 marks

Find and simplify the last term in the expansion of  $(2y - 3x)^7$ .

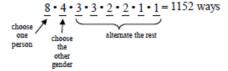
## Solution

Question 6 2 marks

How many different ways can 4 girls and 4 boys be arranged in a row if the girls and the boys must alternate?

#### Solution

#### Method 1



1 mark for beginning with either gender 1 mark for alternating genders

2 marks

#### Method 2

Case 1: 
$$\frac{4 \cdot 4}{B} \cdot \frac{3}{G} \cdot \frac{3}{B} \cdot \frac{3}{G} \cdot \frac{2}{B} \cdot \frac{2}{G} \cdot \frac{1}{B} \cdot \frac{1}{G} = 576$$

1 mark for arrangement of alternating genders

Case 2: 
$$\frac{4}{G} \cdot \frac{4}{B} \cdot \frac{3}{G} \cdot \frac{3}{B} \cdot \frac{2}{G} \cdot \frac{2}{B} \cdot \frac{1}{G} \cdot \frac{1}{B} = 576$$

1/2 mark for two cases

Total number of ways: 576 + 576 = 1152 ways

1/2 mark for addition of cases

2 marks

Question 28 3 marks

Solve the following equation:

$$_{n}P_{2}=_{n}C_{3}$$

### Solution

#### Method 1

$$\frac{n!}{(n-2)!} = \frac{n!}{(n-3)!3!}$$

$$y! (n-3)!3! = y! (n-2)!$$

$$6 = \frac{(n-2)!}{(n-3)!}$$

$$6 = \frac{(n-2)(n-3)!}{(n-3)!}$$

$$1 \text{ mark for substituting for } {n \choose 3}$$

$$1 \text{ mark for simplification}$$

$$1 \text{ mark for expansion of } (n-2)!$$

#### Method 2

$$\frac{n!}{(n-2)!} = \frac{n!}{(n-3)!3!}$$

$$\frac{n!}{(n-2)!} = \frac{n!}{(n-3)!3!}$$

$$\frac{n(n-1)(n-2)!}{(n-3)!6} = \frac{n(n-1)(n-2)(n-3)!}{(n-3)!6}$$

$$\frac{n(n-1)(n-2)!}{(n-3)!6} = \frac{n(n-1)(n-2)(n-3)!}{(n-3)!6}$$

$$\frac{n(n-1)(n-2)!}{(n-3)!6} = \frac{n(n-1)(n-2)(n-3)!}{(n-3)!6}$$
we can divide by  $n$  and  $n-1$  since we know  $n \ge 3$ 

$$6 = \frac{n(n-1)(n-2)}{n(n-1)}$$

$$6 = n-2$$

$$8 = n$$
1 mark for substituting for  $n \ge n$ 

$$\frac{n}{n} = \frac{n}{n}$$

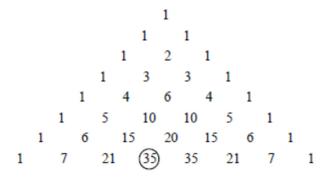
$$\frac{n$$

Question 38 2 marks

Evaluate the coefficient of the term containing  $x^3$  in the expansion of  $(1+x)^7$ . Justify your answer.

## Solution

### Method 1



1 mark for justification

The coefficient of  $x^3$  is 35.

1 mark for identifying coefficient

2 marks

## Method 2

$$t_4 = {}_{7}C_3(1)^4(x)^3$$

$${}_{7}C_3 = \frac{7!}{3!4!}$$

$$= \frac{7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4}!}{\cancel{3}!\cancel{4}!}$$

$$= 35$$

1 mark for justification

1 mark for evaluating coefficient

2 marks

The coefficient of  $x^3$  is 35.

## **June 2013**

## Question 4 (calculator)

3 marks

The 4<sup>th</sup> term in the binomial expansion of  $\left(qx^2 - \frac{3}{x}\right)^{10}$  is 414 720 $x^{11}$ .

Determine the value of q algebraically.

### Solution

$$t_4 = {}_{10}C_3 \left(qx^2\right)^7 \left(-\frac{3}{x}\right)^3 \qquad 2 \text{ marks (1 mark for } {}_{10}C_3, \frac{1}{2} \text{ mark for each consistent factor)}$$

$$414 720x^{11} = 120 \left(q^7x^{14}\right) \left(-\frac{27}{x^3}\right) \qquad \frac{1}{2} \text{ mark for comparing coefficients}$$

$$414 720 = -3240q^7$$

$$q^7 = -128$$

$$q = -2 \qquad \frac{1}{2} \text{ mark for solving for } q$$

$$3 \text{ marks}$$

Question 5 1 mark

Bella has 2 pairs of shoes, 3 pairs of pants, and 10 shirts. Carey has 4 pairs of shoes, 4 pairs of pants, and 4 shirts. An outfit is made up of one pair of shoes, one pair of pants, and one shirt.

Who can make more outfits? Justify your answer.

## Solution

Bella:  $2 \times 3 \times 10 = 60$  outfits

Carey:  $4 \times 4 \times 4 = 64$  outfits

∴ Carey can make more outfits.

1 mark for justification

1 mark

Question 6 2 marks

In the binomial expansion of  $(x - y)^{10}$ , how many terms will be positive? Justify your answer.

## Solution

Six terms will be positive.

1 mark for six terms

The term will be positive when "-y" has an even exponent.

1 mark for justification

2 marks

Question 16 3 marks

Solve algebraically:

$$_{n}C_{2} = 4n + 5$$

## Solution

$$\frac{n!}{(n-2)!2!} = 4n + 5$$

$$\frac{n!}{(n-2)!2!} = 4n + 5$$

$$\frac{n(n-1)(n-2)!}{(n-2)!2!} = 4n + 5$$

$$\frac{n(n-1)(n-2)!}{(n-2)!2!} = 4n + 5$$

$$\frac{n(n-1)}{(n-2)!2!} = 4n + 5$$

$$\frac{n(n-1)}{(n-2)!2!} = 4n + 5$$

$$\frac{n^2}{(n-2)!2!} = 4n + 5$$

$$\frac{n^2}$$

3 marks