# Permutations/Combinations/Binomial Theorem

## January 2014

### Question 4 (calculator) 2 marks

Find and simplify the last term in the expansion of $(2y - 3x)^7$.

**Solution**

$t_8 = {}_7C_7(2y)^0(-3x)^7$

$= -2187x^7$

½ mark for ${}_7C_7$

½ mark for $(2y)^0$

1 mark for $(-3x)^7$ which is equal to $-2187x^7$

**2 marks**

### Question 6 2 marks

How many different ways can 4 girls and 4 boys be arranged in a row if the girls and the boys must alternate?

**Solution**

**Method 1**

$\underline{8} \cdot \underline{4} \cdot \underline{3} \cdot \underline{3} \cdot \underline{2} \cdot \underline{2} \cdot \underline{1} \cdot \underline{1} = 1152$ ways

(8: choose one person; 4: choose the other gender; remaining: alternate the rest)

1 mark for beginning with either gender

1 mark for alternating genders

**2 marks**

**Method 2**

Case 1: $\underset{B}{4} \cdot \underset{G}{4} \cdot \underset{B}{3} \cdot \underset{G}{3} \cdot \underset{B}{2} \cdot \underset{G}{2} \cdot \underset{B}{1} \cdot \underset{G}{1} = 576$

1 mark for arrangement of alternating genders

Case 2: $\underset{G}{4} \cdot \underset{B}{4} \cdot \underset{G}{3} \cdot \underset{B}{3} \cdot \underset{G}{2} \cdot \underset{B}{2} \cdot \underset{G}{1} \cdot \underset{B}{1} = 576$

½ mark for two cases

Total number of ways: $576 + 576 = 1152$ ways

½ mark for addition of cases

**2 marks**

## Question 28 — 3 marks

Solve the following equation:

$$_nP_2 = {}_nC_3$$

### Solution

**Method 1**

$$\frac{n!}{(n-2)!} = \frac{n!}{(n-3)!3!}$$

½ mark for substituting for $_nP_2$

$$\cancel{n!}(n-3)!3! = \cancel{n!}(n-2)!$$

½ mark for substituting for $_nC_3$

$$6 = \frac{(n-2)!}{(n-3)!}$$

1 mark for simplification

$$6 = \frac{(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}}$$

1 mark for expansion of $(n-2)!$

**3 marks**

$$6 = n - 2$$

$$8 = n$$

**Method 2**

$$_nP_2 = {}_nC_3;\ \text{we know } n \geq 3$$

$$\frac{n!}{(n-2)!} = \frac{n!}{(n-3)!3!}$$

½ mark for substituting for $_nP_2$

½ mark for substituting for $_nC_3$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = \frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}6}$$

½ mark for expansion of $_nP_2$

½ mark for expansion of $_nC_3$

$$6 = \frac{\cancel{n}\cancel{(n-1)}(n-2)}{\cancel{n}\cancel{(n-1)}}$$

we can divide by $n$ and $(n-1)$ since we know $n \geq 3$

$$6 = n - 2$$

1 mark for simplification

$$8 = n$$

**3 marks**

## Question 38 2 marks

Evaluate the coefficient of the term containing $x^3$ in the expansion of $(1 + x)^7$.

Justify your answer.

### Solution

**Method 1**

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 1 | | | | | | | |
| | | | | | | 1 | | 1 | | | | | | |
| | | | | | 1 | | 2 | | 1 | | | | | |
| | | | | 1 | | 3 | | 3 | | 1 | | | | |
| | | | 1 | | 4 | | 6 | | 4 | | 1 | | | |
| | | 1 | | 5 | | 10 | | 10 | | 5 | | 1 | | |
| | 1 | | 6 | | 15 | | 20 | | 15 | | 6 | | 1 | |
| 1 | | 7 | | 21 | | (35) | | 35 | | 21 | | 7 | | 1 |

1 mark for justification

The coefficient of $x^3$ is 35.

1 mark for identifying coefficient

**2 marks**

**Method 2**

$$t_4 = {}_7C_3(1)^4(x)^3$$

1 mark for justification

$$\begin{aligned}{}_7C_3 &= \frac{7!}{3!4!} \\ &= \frac{7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4!}}{\cancel{3!}\,\cancel{4!}} \\ &= 35\end{aligned}$$

1 mark for evaluating coefficient

**2 marks**

The coefficient of $x^3$ is 35.

# June 2013

## Question 4 (calculator) 3 marks

The 4th term in the binomial expansion of $\left(qx^2 - \frac{3}{x}\right)^{10}$ is $414\,720x^{11}$.

Determine the value of $q$ algebraically.

**Solution**

$$t_4 = {}_{10}C_3\left(qx^2\right)^7\left(-\frac{3}{x}\right)^3$$

2 marks (1 mark for ${}_{10}C_3$, ½ mark for each consistent factor)

$$414\,720x^{11} = 120\left(q^7x^{14}\right)\left(-\frac{27}{x^3}\right)$$

½ mark for comparing coefficients

$$414\,720 = -3240q^7$$

$$q^7 = -128$$

$$q = -2$$

½ mark for solving for $q$

**3 marks**

## Question 5 1 mark

Bella has 2 pairs of shoes, 3 pairs of pants, and 10 shirts.
Carey has 4 pairs of shoes, 4 pairs of pants, and 4 shirts.
An outfit is made up of one pair of shoes, one pair of pants, and one shirt.

Who can make more outfits? Justify your answer.

**Solution**

Bella: $2 \times 3 \times 10 = 60$ outfits

Carey: $4 \times 4 \times 4 = 64$ outfits

$\therefore$ Carey can make more outfits.

1 mark for justification

**1 mark**

## Question 6 — 2 marks

In the binomial expansion of $(x-y)^{10}$, how many terms will be positive?

Justify your answer.

**Solution**

Six terms will be positive. — 1 mark for six terms

The term will be positive when "$-y$" has an even exponent. — 1 mark for justification

**2 marks**

## Question 16 — 3 marks

Solve algebraically:

$${}_nC_2 = 4n+5$$

**Solution**

$${}_nC_2 = 4n+5$$

$$\frac{n!}{(n-2)!2!} = 4n+5$$

½ mark for factorial notation

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2!} = 4n+5$$

½ mark for factorial expansion
½ mark for simplification of factorial

$$n(n-1) = 2!(4n+5)$$

$$n^2 - n = 8n+10$$

$$n^2 - 9n - 10 = 0$$

½ mark for simplification

$$(n-10)(n+1) = 0$$

$$n = 10 \qquad \cancel{n = -1}$$

½ mark for both values of $n$
½ mark for rejecting extraneous root

**3 marks**