Lesson 3 Remainder Theorem & Factor Theorem

Remainder Theorem:

If $P(x)$ is a polynomial, then $P(a)$ is equal to the remainder when $P(x)$ is divided by $x-a$.

Note:

$x-a$ is a factor of $P(x)$ if the remainder is 0

Ex. 1) Evaluate $f(x) = x^4 - 10x^2 - 2x + 4$ at $f(-3)$.

$$f(-3) = (-3)^4 - 10(-3)^2 - 2(-3) + 4$$
$$= 1 \quad \leftarrow \text{remainder}$$

$\therefore$ By the remainder thm, the remainder is 1 when $f(x)$ is divided by $x+3$

compare
By division

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -10 & -2 & 4 \\ & \downarrow & -3 & 9 & 3 & -3 \\ \hline & 1 & -3 & -1 & 1 & \boxed{1} \end{array} \leftarrow \text{remainder}$$

Ex. 2) Find the remainder, using only the remainder theorem, when $x^4 - 2x^3 + 5x + 2$ is divided by $x + 1$.

$x - a$
$x - (-1)$
$a = -1$

Let $P(x) = x^4 - 2x^3 + 5x + 2$

then $P(-1) = (-1)^4 - 2(-1)^3 + 5(-1) + 2$

$= 0$

$\therefore$ the remainder is 0

which means $x+1$ is a factor of $x^4 - 2x^3 + 5x + 2$

Ex. 3) Determine the value of k when $2x^2 + kx - 5$ is divided by $x - 3$ if the remainder is 7.

$P(3) = 7$

Let $P(x) = 2x^2 + kx - 5$

$a = 3$

sub in

$P(3) = 2(3)^2 + k(3) - 5$

$7 = 18 + 3k - 5$

$-6 = 3k$

$-2 = k$

Ex. 4) When $P(x) = kx^3 + mx^2 + x - 2$ is divided by $x - 1$, the remainder is 6. When $P(x)$ is divided by $x + 2$, the remainder is 12. Determine the values of k and m.

create 2 eqns

① $P(1) = k(1)^3 + m(1)^2 + 1 - 2$

$6 = k + m - 1$

$7 = k + m$ ①

② $P(-2) = k(-2)^3 + m(-2)^2 - 2 - 2$

$12 = -8k + 4m - 4$

$16 = -8k + 4m$

$\div 4$ $\quad 4 = -2k + m$ ②

system of eqns

① − ②

$$\begin{array}{r} 7 = k + \cancel{m} \\ -(4 = -2k + \cancel{m}) \\ \hline 3 = 3k \end{array}$$

elimination method

$k = 1$

sub $k = 1$ into ①

$7 = 1 + m$

$6 = m$

Factor Theorem:

A polynomial, $P(x)$, has a factor $x-a$ iff $P(a)=0$

iff: if and only if

If $P(a)=0$, then $x-a$ is a factor of $P(x)$.

iff ↳ if read thm backwards and forwards, it holds true

Ex. 5) Factor, completely.

$$f(x) = x^3 - 2x^2 - 5x + 6$$

$$\boxed{\textit{possible "a" values} = \frac{\textit{factors of the constant term}}{\textit{factors of the leading coefficient}}}$$

$$= \frac{\text{factors of } 6}{\text{factors of } 1}$$

$$= \frac{\pm1, \pm2, \pm3, \pm6}{\pm1}$$

$$= \pm1, \pm2, \pm3, \pm6$$

$f(x) = x^3 - 2x^2 - 5x + 6$ ← test "a" values using the remainder thm.

$f(-1) = (-1)^3 - 2(-1)^2 - 5(-1) + 6$

$= 8$ $\therefore x+1$ is not a factor since $f(-1) \neq 0$

$f(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$

$= 0$ ✓ $\therefore x+2$ is $\underbrace{\text{a factor of } f(x) \text{ since } f(-2)=0}_{\text{by the factor thm}}$

Now use either long divn or synthetic divn to determine the other factor

$$\begin{array}{r|rrrr} -2 & 1 & -2 & -5 & 6 \\ & \downarrow & -2 & 8 & -6 \\ \hline & 1 & -4 & 3 & \boxed{0} \end{array}$$

$f(x) = (x+2)(x^2 - 4x + 3)$

$\boxed{f(x) = (x+2)(x-3)(x-1)}$

PSF

Ex. 6) Solve.

$$3x^3 - 4x^2 - 5x + 2 = 0$$

poss "a" values : $\dfrac{\text{factors of } 2}{\text{factors of } 3}$

$: \dfrac{\pm 1, \pm 2}{\pm 1, \pm 3}$

$\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$

$P(-1) = 3(-1)^3 - 4(-1)^2 - 5(-1) + 2$

$= 0$

$x+1$ is a factor

$$\begin{array}{r|rrrr} -1 & 3 & -4 & -5 & 2 \\ & \downarrow & -3 & 7 & -2 \\ \hline & 3 & -7 & 2 & \boxed{0} \end{array}$$

$(x+1)(3x^2 - 7x + 2) = 0$

factored form $(x+1)(3x-1)(x-2) = 0$

solve. $x+1=0 \quad 3x-1=0 \quad x-2=0$

$x=-1 \quad x=\frac{1}{3} \quad x=2$

pg 97 # 1a, 2b, 9
4c, 5a, c, h, i

pg 76 # 9c, g, i