One - Sided Limits

Sometimes the values of a fcn, f, tend to different limits as x approaches a number, c, from opposite sides.
When this happens, we call the limit of f as x approaches c from the right, the right hand limit of f at c and the limit as x approaches c from the left, the left hand limit

right hand limit: $\lim_{x \to c^+} f(x)$

left hand limit : $\lim_{x \to c^-} f(x)$

A fcn f(x) has a limit as x approaches c iff the right hand and left hand limits at c exist and are equal.

$$\lim_{x \to c} f(x) = L \iff \lim_{x \to c^+} f(x) = L \text{ and } \lim_{x \to c^-} f(x) = L$$

worksheet 2.2
\# 5, 7
Eval Limits omit \# 8, 9, 11, 13, 14

5 a) 2 b) 3 c) does not exist
d) 4 e) does not exist

7 a) -1 b) -2 c) does not exist
d) 2 e) 0 f) does not exist
g) 1 h) 3

ex

$$f(x) = \begin{cases} \sqrt{1-x^2} & 0 \leq x < 1 \\ 1 & 1 \leq x < 2 \\ 2 & x = 2 \end{cases}$$

a) $\lim_{x \to 1^-} f(x) = 0$

x approaches 1 from the left on the interval $0 \leq x < 1$

$\therefore\ f(1) = \sqrt{1 - 1^2}$

$= 0$

b) $\lim_{x \to 1^+} f(x) = 1$

x approaches 1 from the right on the interval $1 \leq x < 2$

$\therefore\ f(1) = 1$

c) $\lim_{x \to 1} f(x)$

$\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

$\therefore\ \lim_{x \to 1} f(x)$ does not exist