Maxima and Minima

A number c is called the critical value of the fcn $f(x)$ if $f(c)$ is defined and either $f'(c) = 0$ or $f'(c)$ does not exist

Recall factoring:

$$f(x) = x^3 - 6x^2 + 3x + 10$$

poss "a" values are $= \dfrac{\text{factors of constant}}{\text{factors of lead coeff}}$

$= \pm 1, \pm 2, \pm 5, \pm 10$

$$f(2) = 2^3 - 6(2)^2 + 3(2) + 10$$
$$= 0$$

$\therefore x - 2$ is a factor

$$\begin{array}{c|cccc} 2 & 1 & -6 & 3 & 10 \\ & \downarrow & 2 & -8 & -10 \\ \hline & 1 & -4 & -5 & \boxed{0} \end{array}$$

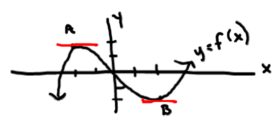
$x^2 - 4x - 5$

$$f(x) = (x-2)(x^2 - 4x - 5)$$
$$f(x) = (x-2)(x-5)(x+1)$$

Recall: At any point $P(x, y)$ the slope of the curve defined by $y = f(x)$ is given by the value of the derivative.

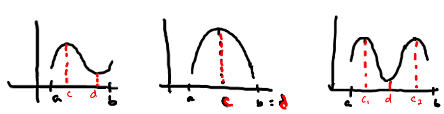
If $\frac{dy}{dx} = 0$ for a single value of x, then the graph of $y = f(x)$ is parallel to the x-axis at the point corresponding to that value of x.

ie



The Extreme Value Theorem

If a graph is continuous on a closed interval [a, b] then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in [a, b]



To find the absolute max and min values of a continuous fcn f on a closed interval [a, b]

1. find the values of f at the critical numbers of f in [a, b]
2. find the values of $f(a)$ and $f(b)$ (endpts)
3. The largest value is the max, smallest the min.

ex.! Find the absolute maximum and minimum values of the function $f(x) = x^3 - 3x^2 + 1$ on the interval $[-\frac{1}{2}, 4]$

critical numbers

$$f(x) = x^3 - 3x^2 + 1$$
$$f'(x) = 3x^2 - 6x$$
$$0 = 3x(x-2)$$
$$x = 0 \quad x = 2$$
$$f'(x) = 0 \text{ @ } \underline{x = 0, 2}$$

$f'(x)$ exists for all x.

$f(0) = 0^3 - 3(0)^2 + 1$
$= \underline{1}$

$f(2) = 2^3 - 3(2)^2 + 1$
$= \underline{-3}$

* sub into $f(x)$, not $f'(x)$

endpts
$[-\frac{1}{2}, 4]$

$f(-\frac{1}{2}) = (-\frac{1}{2})^3 - 3(-\frac{1}{2})^2 + 1$
$= \underline{\frac{1}{8}}$

$f(4) = 4^3 - 3(4)^2 + 1$
$= \underline{17}$

largest is 17
smallest is −3

∴ absolute max is 17
absolute min is −3

since $f(4) = 17 \rightarrow$ pt is (4, 17)
$f(2) = -3 \rightarrow$ pt is (2, −3)

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\# 1, 3, 5, 7, 9,
19, 21, 23