Assignment: Logarithmic Applications

For each of the following use the formula: $A = Pe^{rt}$

A = Amount at time t

P = Original (initial amount)

r = Growth rate

t = time

1. A radioactive substance is decaying. If the initial amount is 10 grams and 6 grams remain after 5 years, then:

   a) Determine the rate of decay

   b) Determine the amount after 10 years

2. It is estimated that 20% of a certain radioactive substance decays in 30 hours.

   a) Determine the rate of decay

   b) What is the half-life of this substance?

3. A biologist studying a colony of bacteria determines that a certain culture grows exponentially such that the bacteria doubles every 4 days.

   a) Determine the rate of growth.

   b) If initially the biologist has 1 200 bacteria present, how many days does it take before 100 000 bacteria are present?

4. The half-life of a radioactive carbon 14 is 5 570 years. If 500 milligrams are present today, determine the amount present after 2 500 years.

5. A radioactive substance decays from 100 units to 40 units in 25 hours. What is the half-life of this substance?

6. Suppose you were able to invest $1 250 in an account where interest was compounded continuously at a rate of 3%. Find the amount after 4 years.

For each of the following use the formula: $M = \log\left(\frac{A}{A_0}\right)$

M = Magnitude of earthquake

A = Intensity of vibrations

A_0 = Intensity of a standard earthquake

7. An earthquake in Vancouver had a magnitude of 6.3 on the Richter scale. An earthquake in Japan had a magnitude of 8.9 on the Richter scale.

   How many times more intense was the Japan earthquake than the Vancouver earthquake?

8. A small tremor of magnitude 2.3 is then followed by a stronger one of magnitude 5.3. How much stronger is the second tremor that the first?

For each of the following use the formula: $PH = -LOG[H^+]$

PH = Acid Strength (Power of Hydrogen)

H^+ = Concentration of hydrogen ion

9. A beaker of acid has a hydrogen concentration of 3.5×10^{-6} mol/L. Calculate the pH of the acid.

10. If a beaker of acids has a pH of 4.1, calculate the hydrogen concentration of the acid.

11. A beaker of acid has a pH of 4.9, and a second beaker has a pH of 7.6. Determine how many times higher the hydrogen concentration is in the strong acid as compared to the weaker one.

For each of the following use the formula: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

A = Compound amount (new investment value

P = Original amount

r = interest rate (as a decimal)

n = # of Compounding periods

t = time

12. Find the new investment of:

a) $2 000 at 8% compounded annually over 15 years

b) $5 000 at 12% compounded monthly over 4 years

c) $15 000 at 6% compounded semi-annually over 7 years

13. You plan to buy a car and have saved $5 000. The car you want costs $5 900. How long will you have to invest your money if you can invest at 6.12% compounded quarterly?

14. The Robinsons plan to invest money for their newborn son so that he has $30 000 available for university on his 18th birthday. Assuming a growth rate of 7% compounded semi-annually, how much will the Robinson's need to invest today?

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15. The amount of a chemical in grams that will dissolve in a solution is given by $C = 8e^{0.3t}$ where t is the temperature in Celsius of the solution. Find t when $C = 100$ grams.

16. Determine how many monthly investments of $50 would have to be deposited into a savings account that pays 3% annual interest, compounded monthly, for the account's future value to be $50 000.

$FV = \frac{R[(1+i)^n - 1]}{i}$

FV = Present Value

R = investment payment

i = $\frac{annual\ interest\ rate}{\#\ of\ compounding\ periods\ per\ year}$

n = # of investments

17. A person borrows $15 000 to buy a car. The person can afford to pay $300 a month. The loan will be repaid with equal monthly payments at 6% annual interest, compounded monthly. How many monthly payments will the person make?

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

PV = Present Value

R = payment

i = $\frac{annual\ interest\ rate}{\#\ of\ compounding\ periods\ per\ year}$

n = # of payments

Answers

1. a) -0.1022 b) 3.6 g
2. a) -0.0074 b) 93.2 hours
3. a) 0.1733 b) 26 days
4. r = -0.000124, A = 366.3 mg
5. r = -0.03665, t = 18.9 hours
6. $1 409.37
7. Vancouver – $10^{6.3}$, Japan – $10^{8.9}$, Japan is 398.1 times as intense
8. Strong – $10^{5.3}$, Small – $10^{2.3}$, Strong tremor is 1 000 times as intense
9. 5.46
10. 7.94×10^{-5} mol/L
11. 501
12. a) $6 344.34 b) $8 061.13 c) $22 688.85
13. 2.7 years
14. $8 694.98
15. 8.42°C
16. 502 monthly payments
17. 58 payments