Pre-Calculus 12 Laws of Logarithms

Let "n" be any real number, and M, N and "a" are positive real numbers with $a \neq 1$

Name of Law	Law	Example
Product Law	$\log_a MN = \log_a M + \log_a N$	$\log 6 = \log 2 + \log 3$
Quotient Law	$\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$	$\log \frac{3}{4} = \log 3 - \log 4$
Power Law	$\log_a M^x = x \log_a M$	$\log 5^2 = 2 \log 5$

Ex. 1) Simplify the expression. Use a calculator to verify.

$\log 7 + \log 8 \longrightarrow 1.748$

product law $\log(56) \longrightarrow 1.748$

Ex. 2) Write each as a single logarithm

a) $\log x + 3 \log y$ power law $\log x + \log y^3$

product law $\log(xy^3)$

Try

$2 \log x + \log 3y$ $\log(x^2 3y)$ $\log(3x^2 y)$

$\log y^3 - \log x$

$\log\left(\frac{y^3}{x}\right)$

b) $\log x + 2 \log y - 4 \log z$

$\log x + \log y^2 - \log z^4$

$\log\left(\frac{xy^2}{z^4}\right)$ quotient law

c) $\log_2 6 - 3$ ← change to a log

$\log_2 6 - 3 \log_2 2$ ← $\log_2 2 = 1$

$\log_2 6 - \log_2 2^3$ ← power law

$\log_2 6 - \log_2 8$

$\log_2\left(\frac{6}{8}\right)$

$\log_2\left(\frac{3}{4}\right)$

Ex. 3) Expand, using the laws of logarithms.

a) $\log\left(\frac{x}{y^2}\right)$

$\log x - \log y^2$ ← quotient law

$\log x - 2\log y$ — power law

b) $\log\left(\frac{x^2 y^{1/2}}{z}\right)$ If given as $\sqrt{y}$, change to $y^{\frac{1}{2}}$

$\log x^2 + \log y^{\frac{1}{2}} - \log z$ Each term has its own log

$2\log x + \frac{1}{2}\log y - \log z$

Ex. 4) Evaluate:

a) $3\log_9 6 - \log_9 72$

$\log_9\left(\frac{6^3}{72}\right)$ Use log laws

$\log_9\left(\frac{216}{72}\right)$

$\log_9 3$ simplify

$\frac{1}{2}$ since $9^{\frac{1}{2}} = 3$, $\log_9 3 = \frac{1}{2}$ $(\sqrt{9})$

b) $2\log_4 6 - 3\log_4 3 + \log_4 12$

$\log_4 6^2 - \log_4 3^3 + \log_4 12$

$\log_4 6^2 + \log_4 12 - \log_4 3^3$

$\log_4\left(\frac{6^2 \cdot 12}{3^3}\right)$

$\log_4 16$

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Prop of Logs
4-12 even
13-21 odd

Morgan

Try

$\log\left(\frac{xy}{z^2}\right)$

$\log x + \log y - 2\log z$

$\log\left(\frac{x^{\frac{1}{3}}z}{y^3}\right)$

$\frac{1}{3}\log x + \log z - 3\log y$

Ex. 5) Given $\log_a 2 = 0.3562$, $\log_a 3 = 0.5646$ and $\log_a 5 = 0.8271$, use the laws of logarithms to evaluate.

a.) $\log_a 15$ ← use a product, quotient or power of 2, 3 and/or 5

$\log_a (3 \cdot 5)$

$\log_a 3 + \log_a 5$

sub values → $0.5646 + 0.8271$

1.3917

b.) $\log_a 27$

$\log_a 3^3$

$3 \log_a 3$

$3 (0.5646)$

1.6938

pg 400
1a, 2a, b
3a, 8a, d
11a, 12a