

Key

Linear Functions

Key Ideas:

1. Slopes

- Label your points $(x_1, y_1), (x_2, y_2)$
- $m = \frac{\text{rise}}{\text{run}}$, (build a triangle)
- Slope Formula
- Rate of Change
- Parallel (same), Perpendicular (negative reciprocals – “flip and switch”)

2. Graphing Linear Functions

- Slope-Intercept Form ($y = mx + b$)
- Point-Slope Form ($y - y_1 = m(x - x_1)$)
- Graph using Intercepts

3. Forms of the Equation

- Slope-Intercept form ($y = mx + b$)
- Slope-Point form ($y - y_1 = m(x - x_1)$)
- General form (no fractions or decimals, and x is always positive), ($Ax + By + C = 0$)

4. Writing Equations

- Find the slope
 - i. Use the slope formula
 - ii. Re-write the equation in slope-intercept form
- Find a point
- Substitute the slope and point into point-slope formula
- Re-write point-slope formula into general form or slope-intercept form if needed

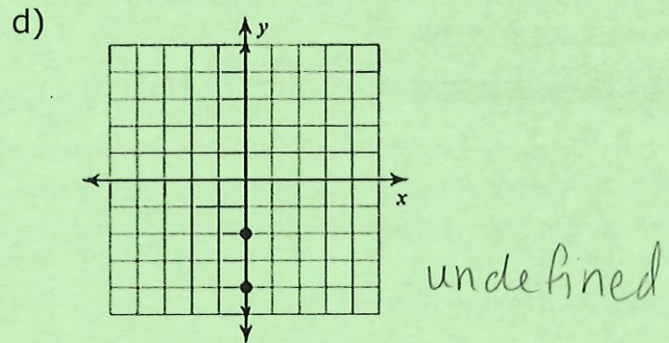
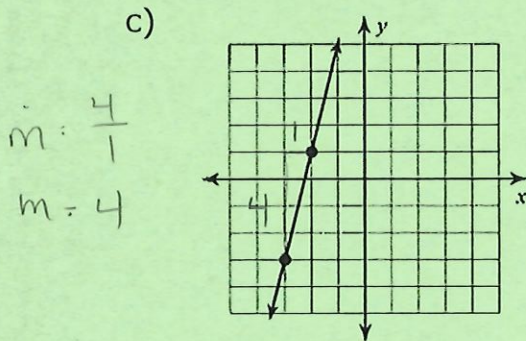
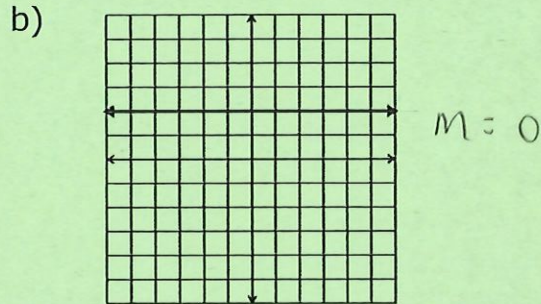
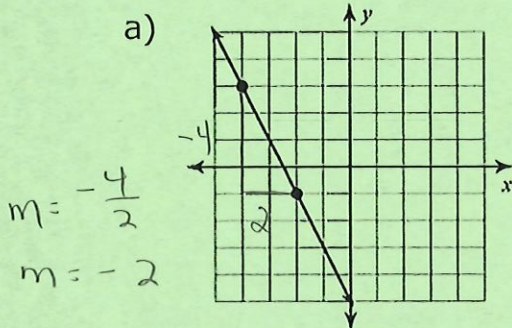
5. Distance and Midpoint

- Label your points $(x_1, y_1), (x_2, y_2)$
- Use “Frank” to find the formulas
- Make sure the answer for the midpoint is written as a coordinate

***Note:** Substitute into formulas carefully! Watch your positive and negative signs!!

Slope of a Line

1. Determine the slope of the line using Rise over Run (Build the triangle).



2. Determine the slope of a line through the following points.

a) x_1, y_1, x_2, y_2
 $(3, -7), (-5, -10)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-10 - (-7)}{-5 - 3}$$

$$= \frac{-3}{-8}$$

$$m = \frac{3}{8}$$

b) x_1, y_1, x_2, y_2
 $(-3, 8), (-1, 3)$

$$m = \frac{3 - 8}{-1 - (-3)}$$

$$m = \frac{-5}{2}$$

3. Determine the value of "x" if the slope running through the points $(0, -2)$ and $(x, 4)$ has a value of 2.

x_2, y_2

m

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$2 = \frac{4 - (-2)}{x - 0}$$

$$2x = 6$$

$$x = 3$$

4. Determine the value of "y" if the slope running through the points $(6, y)$ and $(1, -5)$ is $-\frac{2}{3}$.

x_1, y_1

x_2, y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-\frac{2}{3} = \frac{-5 - y}{1 - 6}$$

~~$$-\frac{2}{3} = \frac{-5 - y}{-5}$$~~

$$10 = 3(-5 - y)$$

$$10 = -15 - 3y$$

$$25 = -3y$$

$$\frac{25}{-3} = y$$

5. The coordinates of two points on two different lines are given. Are the two lines parallel, perpendicular, or neither? Justify your choice.

- a) $W(-3, 5), X(8, 3)$ and $C(6, 6), D(1, 8)$

$$m_{WX} = \frac{3 - 5}{8 - (-3)}$$

$$= \frac{-2}{11}$$

$$m_{CD} = \frac{8 - 6}{1 - 6}$$

$$= \frac{2}{-5}$$

neither, not equal nor negative reciprocals

- b) $J(3, -4), K(9, 2)$ and $P(5, -4), Q(2, -1)$

$$m_{JK} = \frac{2 - (-4)}{9 - 3}$$

$$= \frac{6}{6}$$

$$= 1$$

$$m = \frac{-1 - (-4)}{2 - 5}$$

$$= \frac{3}{-3}$$

$$= -1$$

perpendicular, negative reciprocals

- c) $R(-3, 2), S(1, -6)$ and $E(-2, 1), F(-5, 7)$

$$m = \frac{-6 - 2}{1 - (-3)}$$

$$= \frac{-8}{4}$$

$$= -2$$

$$m = \frac{7 - 1}{-5 - (-2)}$$

$$= \frac{6}{-3}$$

$$= -2$$

parallel, equal

6. Determine the slope of a line **parallel and perpendicular** to: $3x - 5y + 10 = 0$.

$$\parallel m = \frac{3}{5}$$

$$\perp m = -\frac{5}{3}$$

$$3x + 10 = 5y$$

$$\frac{3}{5}x + 2 = y$$

7. A line through the points $(5, y)$ and $(-6, -3)$ is perpendicular to a line with a slope of $\frac{1}{4}$. Determine the value of "y".

$$\perp m = -4$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-4 = \frac{-3 - y}{-6 - 5}$$

$$-4 = \frac{-3 - y}{-11}$$

$$44 = -3 - y$$

$$47 = -y$$

$$-47 = y$$

8. A line through the points $(-4, 1)$ and $(x, -2)$ is parallel to the line $5x - 3y + 10 = 0$. Determine the value of "x".

$$\frac{5}{3} = \frac{-2 - 1}{x + 4}$$

$$5x + 20 = -9$$

$$5x = -29$$

$$x = -\frac{29}{5}$$

$$5x + 10 = 3y$$

$$\frac{5}{3}x + \frac{10}{3} = y$$

$$\parallel m = \frac{5}{3}$$

9. The following represents the graph for a helium balloon's flight.

- a) Determine the rate of change of the graph.

$$\frac{1000 \text{ ft}}{1 \text{ min}}$$

$$1000 \text{ ft/min}$$

- b) What does this slope (rate of change) mean?

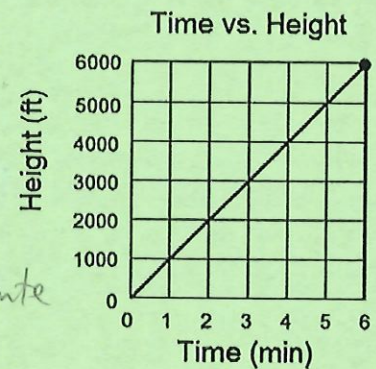
The balloon's height rises 1000 ft per minute

- c) When is the balloon at 5 000 ft?

$$5 \text{ min}$$

- d) How high is the balloon off the ground at 2 minutes?

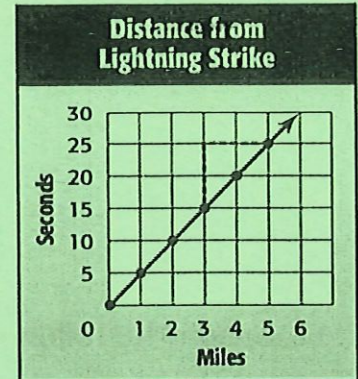
$$2000 \text{ ft}$$



10. The following graph represents the relationship between the number of seconds it takes to hear the thunder after a lightning strike and the distance you are from the lightning.

a) Determine the rate of change for the graph.

5 secs/mile



b) What does this rate of change mean?

For every 5 seconds it takes to hear the thunder after a lightning strike, you are 1 mile away from the lightning strike

c) Determine the number of miles you are from a lightning strike if the time between the strike and thunder was 15 seconds.

3 miles

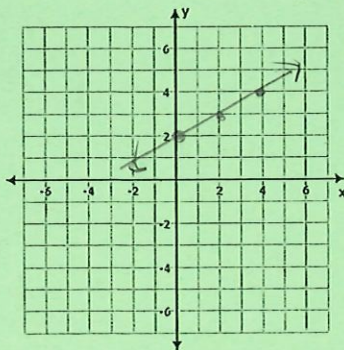
Graphing Linear Functions

11. Graph the following using the Slope-Intercept method for graphing.

a) $-x + 2y = 4$

$$2y = x + 4$$

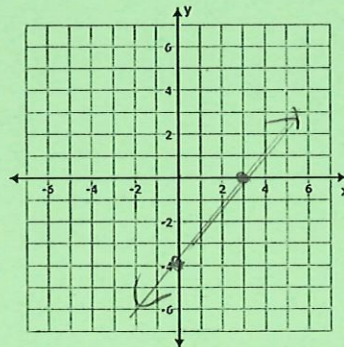
$$y = \frac{1}{2}x + 2$$



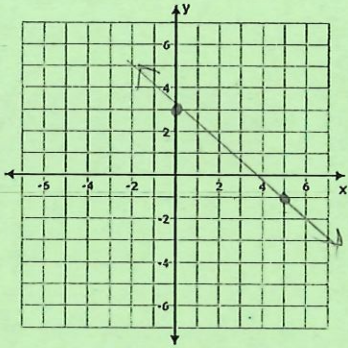
b) $4x - 3y = 12$

$$-3y = -4x + 12$$

$$y = \frac{4}{3}x - 4$$



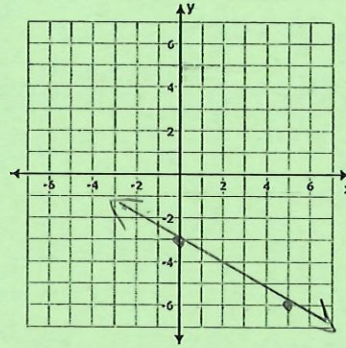
c) $y = -\frac{4}{5}x + 3$



d) $-3x - 5y = 15$

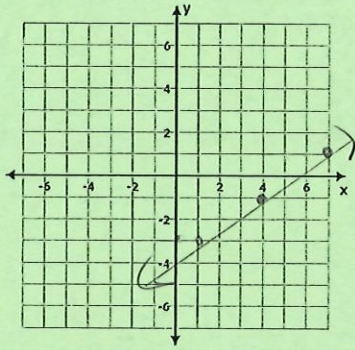
$$-5y = 3x + 15$$

$$y = -\frac{3}{5}x - 3$$

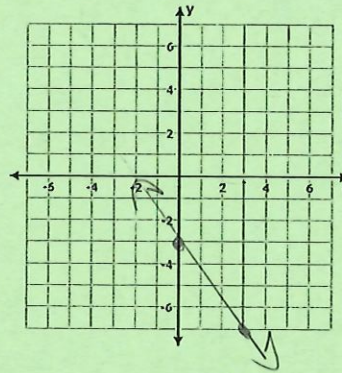


12. Graph the following using the Point-Slope method for graphing.

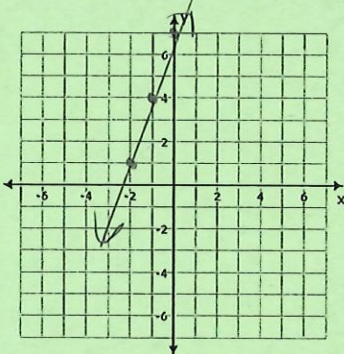
a) $y + 1 = \frac{2}{3}(x - 4)$ pt (4, -1)



b) $y + 3 = -\frac{4}{3}x$ pt (0, -3)

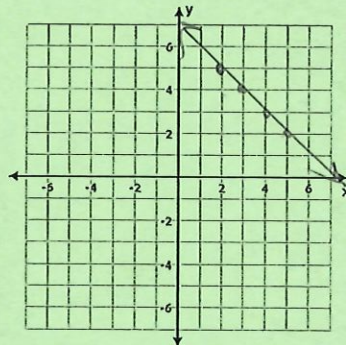


c) $y - 4 = 3(x + 1)$ pt (-1, 4)



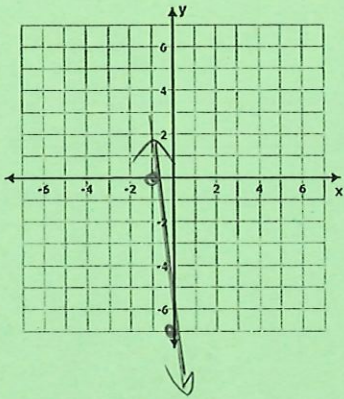
d) $y - 5 = -(x - 2)$

pt (2, 5)



13. Graph the following by finding the intercepts of the graph.

a) $7x + y + 7 = 0$



$$x = 0$$

$$7(0) + y + 7 = 0$$

$$y = -7$$

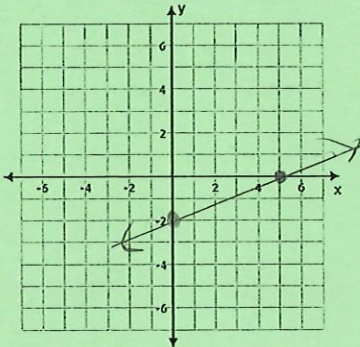
$$y = 0$$

$$7x + 0 + 7 = 0$$

$$7x = -7$$

$$x = -1$$

b) $2x - 5y - 10 = 0$



$$x = 0$$

$$2(0) - 5y - 10 = 0$$

$$-5y = 10$$

$$y = -2$$

$$y = 0$$

$$2x - 5(0) - 10 = 0$$

$$2x = 10$$

$$x = 5$$

Writing Equations of Linear Functions

14. Write each equation in general form.

a) $y = \frac{5}{4}x - \frac{3}{5}$

$$20y = 5(5)x - 4(3)$$

$$20y = 25x - 12$$

$$0 = 25x - 20y - 12$$

b) $y + 4 = \frac{2}{3}(x - 1)$

$$3(y + 4) = 2(x - 1)$$

$$3y + 12 = 2x - 2$$

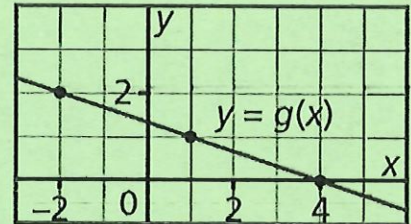
$$0 = 2x - 3y - 14$$

$$y - y_1 = m(x - x_1)$$

15. a) Write an equation in slope-point form for this line.

$$m = -\frac{1}{3} \quad \text{pt } (1, 1)$$

$$y - 1 = -\frac{1}{3}(x - 1)$$



b) Write the equation in part a) in slope-intercept form. What is the y-intercept of this line?

$$y - 1 = -\frac{1}{3}x + \frac{1}{3} + 1$$

$$y = -\frac{1}{3}x + \frac{4}{3}$$

$$\uparrow \\ \text{y-int} = \frac{4}{3}$$

16. Write an equation for a line in **general form** having: **slope** of $-\frac{3}{5}$, **y-intercept** of -8 .

$$y = -\frac{3}{5}x - 8$$

$$5y = -3x - 40$$

$$3x + 5y + 40 = 0$$

17. Write an equation for a line having: **slope** of 4, passing through $P(-2, 9)$.

$$y - 9 = 4(x + 2)$$

18. Write an equation for a line in **slope-intercept form**: passing through $A(5, -1)$ and $B(-3, 4)$.

$$m = \frac{4 - (-1)}{-3 - 5}$$

$$= \frac{5}{-8}$$

$$y + 1 = -\frac{5}{8}(x - 5)$$

$$y + 1 = -\frac{5}{8}x + \frac{25}{8} - 1$$

$$y = -\frac{5}{8}x + \frac{17}{8}$$

19. Write in **general form**: $y - 2 = \frac{-2}{3}(x + 6)$.

$$3y - 6 = -2x - 12$$

$$2x + 3y + 6 = 0$$

20. Write in **general form: parallel** to $5y = 3x - 1$, passing through $(8, -2)$.

$$y = \frac{3}{5}x - 1$$

$$\parallel m = \frac{3}{5}$$

$$y + 2 = \frac{3}{5}(x - 8)$$

$$5y + 10 = 3x - 24$$

$$0 = 3x - 5y - 34$$

21. Write in **general form: perpendicular** to $3x - 4y = 12$, passing through $(-4, 3)$.

$$y - 3 = -\frac{4}{3}(x + 4)$$

$$3y - 9 = -4x - 16$$

$$4x + 3y + 7 = 0$$

$$-4y = -3x + 12$$

$$y = \frac{3}{4}x - 3$$

$$\perp m = -\frac{4}{3}$$

Distance and Midpoint

22. Calculate the **distance** from $A(6, -4)$ to $B(1, 5)$:

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(1 - 6)^2 + (5 - (-4))^2} \\
 &= \sqrt{(-5)^2 + 9^2} \\
 &= \sqrt{25 + 81}
 \end{aligned}
 \quad d = \sqrt{106}$$

23. Given $\triangle ABC$ with $A(0, -2)$, $B(4, 5)$, and $C(-2, 1)$.

a) Classify the triangle as scalene, isosceles or equilateral.

$$\begin{aligned}
 d_{AB} &= \sqrt{(4 - 0)^2 + (5 - (-2))^2} \\
 &= \sqrt{16 + 49} \\
 &= \sqrt{65}
 \end{aligned}
 \quad
 \begin{aligned}
 d_{AC} &= \sqrt{(0 - (-2))^2 + (-2 - 1)^2} \\
 &= \sqrt{4 + 9} \\
 &= \sqrt{13}
 \end{aligned}$$

$$\begin{aligned}
 d_{BC} &= \sqrt{(4 - (-2))^2 + (5 - 1)^2} \\
 &= \sqrt{36 + 16} \\
 &= \sqrt{52}
 \end{aligned}
 \quad \therefore \text{scalene}$$

b) Determine the length of median AM , where M is the midpoint of BC .

$$\begin{aligned}
 M_{BC} &= \left(\frac{4 + (-2)}{2}, \frac{5 + 1}{2} \right) \\
 &= (1, 3)
 \end{aligned}
 \quad A(0, -2)$$

$$\begin{aligned}
 d &= \sqrt{(0 - 1)^2 + (-2 - 3)^2} \\
 &= \sqrt{1 + 25} \\
 &= \sqrt{26}
 \end{aligned}$$

24. Determine the **midpoint** of $A(-5, 3)$ and $B(-11, -7)$.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$M = \left(\frac{-5 + (-11)}{2}, \frac{3 + (-7)}{2} \right)$$

$$= (-8, -2)$$

25. Determine the coordinates of A, given that $M(-1, 2)$ is the midpoint of AB and B has coordinates $(4, 7)$.

$$x_m = \frac{x_1 + x_2}{2} \qquad y_m = \frac{y_1 + y_2}{2}$$

$$-1 = \frac{4 + x_2}{2} \qquad 2 = \frac{7 + y_2}{2}$$

$$-2 = 4 + x_2 \qquad 4 = 7 + y_2$$

$$-6 = x_2 \qquad -3 = y_2$$

26. The endpoints of the diameter of a circle are $(-4, -2)$ and $(6, 4)$. Determine the coordinates of the centre of the circle.

$$M = \left(\frac{-4 + 6}{2}, \frac{-2 + 4}{2} \right)$$

$$= (1, 1)$$