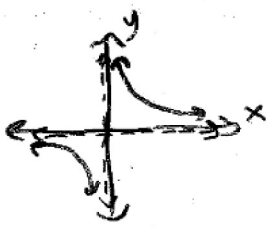
Limits Involving Infinity

ex. 1:



$y = \frac{1}{x}$

As x approaches ∞, $\frac{1}{x} \to 0$

$\therefore \lim_{x \to \infty} \left(\frac{1}{x}\right) = 0$

Also note H.A is $y = 0$

As $x \to -\infty$, $\frac{1}{x} \to 0$

$\therefore \lim_{x \to -\infty} \left(\frac{1}{x}\right) = 0$

Recall:

Horizontal asymptotes

$f(x) = \frac{p(x)}{q(x)}$

m = degree of the numerator, $p(x)$

n = degree of the denominator, $q(x)$

$m < n$	the H.A. is $y = 0$
$m = n$	the H.A. is $y = \frac{a}{b}$ > lead coeff.
$m > n$	no H.A. exists

The line $y=b$ is a horizontal asymptote of the graph of a fcn. $y=f(x)$ if either

$$\lim_{x\to\infty} f(x)=b \quad \text{or} \quad \lim_{x\to-\infty} f(x)=b$$

We can also use the degrees of the numerator and denominator to find limits as $x \to \pm\infty$

ex. 2 Find the limits

a) $\lim_{x\to\infty} \frac{3x^2+2x-1}{x+1}$ $m > n$ H.A. no H.A.

doesn't exist

sometimes we say it is ∞

b) $\lim_{x\to\infty} \frac{3x-1}{4x+2}$ $m=n$ H.A. $y=\frac{3}{4}$

$$\lim_{x\to\infty}\left(\frac{3x-1}{4x+2}\right)=\frac{3}{4}$$

c) $\lim_{x\to\infty} \frac{2}{x-1}$ $m<n$ H.A. $y=0$

$$\lim_{x\to\infty}\left[\frac{2}{x-1}\right]=0$$

Evaluate

$$\lim_{x\to\infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1}$$

$$\lim_{x\to\infty} \frac{\frac{3x^2 - x - 2}{x^2}}{\frac{5x^2+4x+1}{x^2}}$$

$\div x^2$ highest degree

$$\lim_{x\to\infty} \frac{\frac{3x^2}{x^2} - \frac{1}{x^2} - \frac{2}{x^2}}{\frac{5x^2}{x^2} + \frac{4x}{x^2} + \frac{1}{x^2}}$$

$2 \cdot \frac{1}{x^2}$

$c \lim f(x)$

$$\frac{\lim_{x\to\infty} 3 - \lim_{x\to\infty} \frac{1}{x} - 2\lim_{x\to\infty} \frac{1}{x^2}}{\lim_{x\to\infty} 5 + 4\lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \frac{1}{x^2}}$$

Prop 5 & 4

$$\frac{3 - 0 - 0}{5 + 0 + 0}$$

$$\frac{3}{5}$$

#1-6
13-22
27
33
~~44~~

Eval Limits
#1, 3, 7, 8, 10

3) $\lim_{x \to \infty} \frac{3x + \sin x^2}{x^2}$

$3 \lim_{x \to \infty} \frac{1}{x^2} + \lim_{x \to \infty} \frac{\sin x^2}{x^2}$

$0 \qquad +$