Transformations Lesson 3 Stretches and Compressions

Monday, February 6, 2023 11:58 AM

Transformations Lesson 3 Stretches and Compressions

Lesson 3 Stretches/Compressions

Stretches/Compressions change the shape of the graph but do not change the orientation of the graph.

Graphs of the form $y = af(x)$ such as:

$y = 3f(x)$
$y = 2f(x)$
} stretch vertically

$y = f(x)$ parent graph

$y = \frac{1}{2}f(x)$
$y = \frac{1}{4}f(x)$
} compress vertically

Vertical Stretches/Compressions and Reflections

$y = af(x)$ is the image graph of $y = f(x)$ after a vertical stretch/compression or reflection.

Note: Point (x, y) becomes point (x, ay)

- When $a > 1$, there is a vertical stretch
- When $0 < a < 1$ there is a vertical compression
- When $a < 0$, there is a reflection in the x-axis as well as a stretch or compression

negative

Graphs of the form $y = f(bx)$

$y = f(4x)$
$y = f(2x)$
} stretched horizontally by a factor of $\frac{1}{b}$
* Divide x-coordinates by b

$y = f(x)$ parent graph

$y = f\left(\frac{1}{2}x\right)$
$y = f\left(\frac{1}{4}x\right)$
} stretched horizontally by a factor of $\frac{1}{b}$
* Divide x-coordinates by b
or multiply by reciprocal of b

Horizontal Stretches/Compressions and Reflections

$y = f(bx)$ is the image graph of $y = f(x)$ after a horizontal stretch, compression or reflection.

Note: Point (x, y) becomes point $\left(\frac{x}{b}, y\right)$

- When $b > 1$, there is a horizontal compression
- When $0 < b < 1$, there is a horizontal stretch
- When $b < 0$ (negative), there is a reflection in the y-axis as well as the stretch or compression

Ex. 1) Sketch

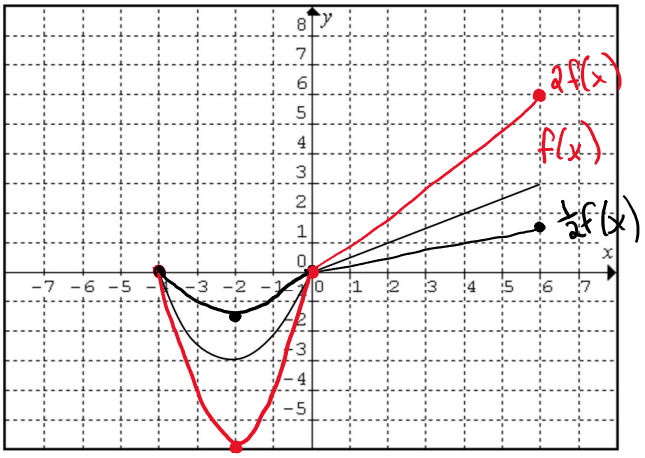
a) $2f(x)$

mult y-values by 2

x	f(x)	2f(x)	$\frac{1}{2}f(x)$
-4	0	0	0
-2	-3	-6	$-1\frac{1}{2}$
0	0	0	0
6	3	6	$1\frac{1}{2}$

b) $\frac{1}{2}f(x)$

plot



Ex. 2) Given $f(x) = |x|$, sketch

a) $g(x) = f(3x)$

divide x-coords by 3

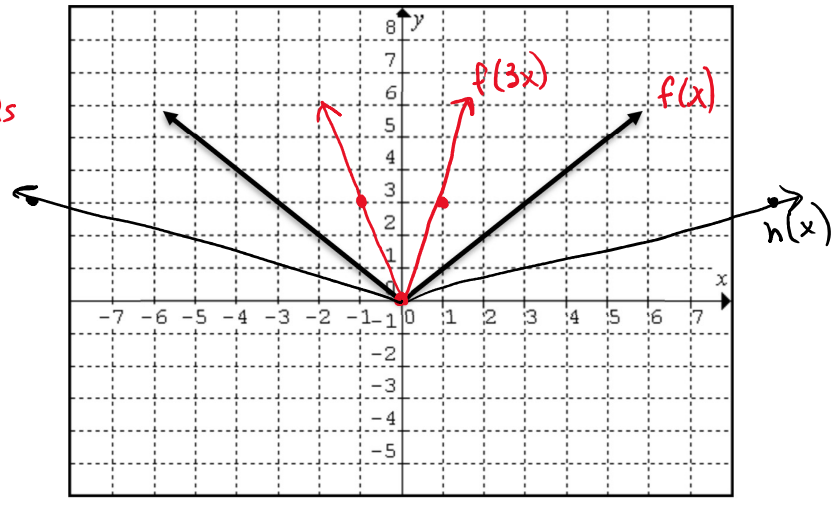
x	f(x)	f(3x)	
-3	3	-1	(-1, 3)
0	0	0	(0, 0)
3	3	1	(1, 3)

b) $h(x) = f(\frac{1}{3}x)$

$(-3, 3) \rightarrow (-9, 3)$

$(0, 0) \rightarrow (0, 0)$

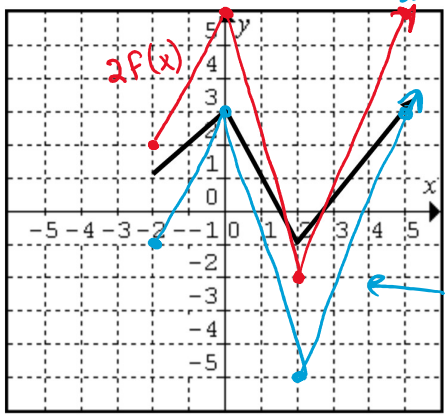
$(3, 3) \rightarrow (9, 3)$



Ex. 3) Sketch $g(x) = 2f(x) - 3$

mult y-coords by 2

down 3



* stretches/reflections always done first!

Ex. 4) Given the function $f(x) = 2x - 1$, determine the equation of:

a) $y = 2f(x)$

$y = 2(2x - 1)$

or

$y = 4x - 2$

b) $y = f(3x)$

replaced x

$y = 2x - 1$

$y = 2(3x) - 1$

or

$y = 6x - 1$

Ex. 5) The x-intercepts (zeros) of the graph $f(x)$ are -2, 4, and 6. Determine the x-intercepts after the following transformations.

a) $y = \frac{1}{2}f(x)$

mult y-coords by $\frac{1}{2}$

-2, 4, 6

no change

b) $y = f(-2x)$

* divide by (-2)

1, -2, -3

Ex. 6) Determine the equation of $g(x)$ after the following transformations of $f(x)$.

- Horizontal stretch by a factor of 2 $b = \frac{1}{2}$
- Vertical compression by a factor of $\frac{1}{3}$ $a = \frac{1}{3}$
- Reflection in the x-axis $a = -\frac{1}{3}$

Recall $y = af(bx)$

$g(x) = -\frac{1}{3} f\left(\frac{1}{2}x\right)$

Bulawka's Bullets

☺ Remember divide by b, multiply by a

☺ Recall: $f(x)$ often replace y, so $af(x)$ is similar to ay which tells you to multiply y-coordinates by a

☺ b must be factored out before performing horizontal tranlations

Assign

1a, c

3, 4, 5, 6a, b, 7*, 9