Pre-Calculus 12
Solving Trig Equations Algebraically

Ex. 1) Solve $\sin x = -\frac{\sqrt{3}}{2}$ *for* $0 \le x \le 2\pi$

looking for y-coords of $-\frac{\sqrt{3}}{2}$

(x, y)

$(\cos x, \sin x)$

mult of $\frac{\pi}{3}$

$(\frac{1}{2}, \frac{\sqrt{3}}{2})$

$\sin x < 0$ in Quads III, IV

$x = \frac{4\pi}{3}, \frac{5\pi}{3}$

Ex. 2) Solve

a) $7 + 2\sin x = 4\sin x + 5$ for $-360° < x \le 0$

Combine like terms

Isolate $\sin x$

$2 = 2\sin x$

$1 = \sin x$

$x_r = 90°$

$x_c = 90° - 360°$

$x_c = -270°$

b) Determine the general solution for the above equation.

$x = 90° \pm 360°k \quad , k \in \mathbb{Z}$

Ex. 3) Solve $1 + 2\cos x = 5\cos x$ for $-\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

$-1.57 \le x \le 4.71$

$1 = 3\cos x$

$\frac{1}{3} = \cos x$

not a special circle value $\longrightarrow$ calc question

$x_r = \cos^{-1}\left(\frac{1}{3}\right)$

$x_r = 1.2309...$

* calc in radians to match gn interval

$\cos x > 0$ in QI, QIV

QI $x = x_r = 1.231$

QIV $x = 2\pi - x_r = 2\pi - 1.2309... = 5.052$ ← not in interval

$x_c = 5.052 - 2\pi = -1.231$

Ex. 4) Solve $2\cos^2 x = 1$ for $0° \le x \le 360°$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$ ← rationalized form $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$ ← on special circle

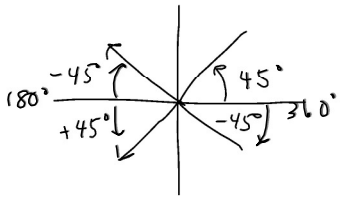
all 4 Quads

QI $x = 45°$

QII $x = 135°$

QIII $x = 225°$

QIV $x = 315°$



Ex. 5) Solve $\sin^2 x - 2\sin x = 0$ for $0 \le x < 2\pi$

GCF $\sin x$

$\sin x(\sin x - 2) = 0$

$\sin x = 0$ $\quad \sin x - 2 = 0$

$x = 0, \pi, 2\pi$ (not in interval)

$\sin x = 2$ No sol'n

$-1 \le \sin x \le 1$

Ex. 6) Solve $\tan^2\theta - 5\tan\theta + 4 = 0$ *for* $\theta \in \mathbb{R}$ — in radians, general sol'n

PSF
P 4
S -5
F -4, -1

$(\tan\theta - 4)(\tan\theta - 1) = 0$

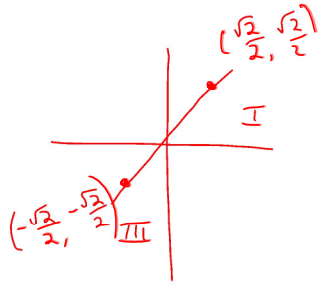
$\tan\theta - 4 = 0$
$\tan\theta = 4$
$\theta_r = \tan^{-1}(4)$
$\theta_r = 1.32581...$

$\tan\theta > 0$ in QI, QIII

$\theta = 1.326$

QIII $\theta = \pi + 1.32581...$
$= 4.467$

$\tan\theta - 1 = 0$
$\tan\theta = 1$
$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$



If not factorable, use the quadratic formula (on your formula sheet)

General sol'n

$\theta = 1.326 + k\pi, \quad k \in \mathbb{Z}$

$\theta = \frac{\pi}{4} + k\pi, \quad k \in \mathbb{Z}$

OR

$\theta = 1.326 \pm 2k\pi$
$\theta = 4.467 \pm 2k\pi$
$\theta = \frac{\pi}{4} \pm 2k\pi$
$\theta = \frac{5\pi}{4} \pm 2k\pi$

$k \in \mathbb{Z}$

Worksheet
2a, 5
Pg 212 #7a, b, 16

Review
pg. 215
Practice Test
pg 218 #1-12

Assignment: Pg. 593; #6b, 7b, 10, 11a, 12a, 13, 14b