Lesson 5 Inverse Relations

An inverse relation is a relation that has been reflected in the line $y = x$.

Notation:

- $y = f^{-1}(x)$
- $x = f(y)$

When the inverse is a function we use $y = f^{-1}(x)$ instead of $x = f(y)$

↳ passes v.l.t.

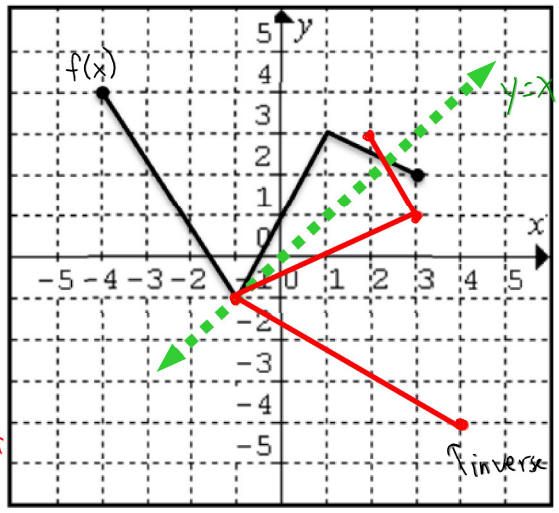
Ex. 1) Given the graph of $y = f(x)$,

a) sketch the graph of its inverse.

switch x and y values

$y = f(x)$	inverse
$(-4, 4)$	$(4, -4)$
$(-1, -1)$	$(-1, -1)$
$(1, 3)$	$(3, 1)$
$(3, 2)$	$(2, 3)$

invariant point →



b) explain why the inverse is not a function.

No, it fails the vertical line test

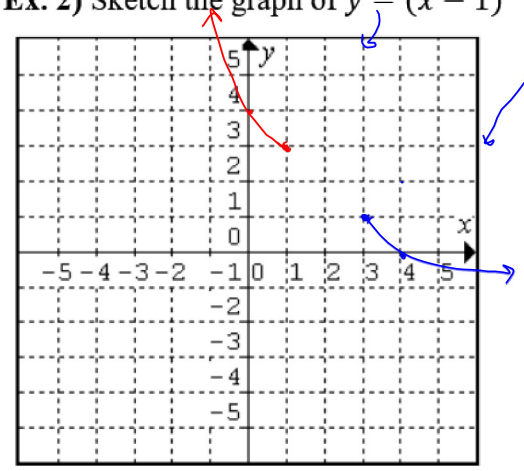
(There is more than one value of y for multiple values of x)

c) state the domain and range of the function and its inverse.

	$f(x)$	inverse
D:	$[-4, 3]$	$[-1, 4]$
R:	$[-1, 4]$	$[-4, 3]$

The properties of x and y also switch

Ex. 2) Sketch the graph of $y = (x-1)^2 + 3$ and its inverse.



∴ $x \geq 1$

↳ Range of inverse

$y \geq 1$

a) Is the inverse graph a function? No

b) Determine two ways to restrict the domain of $y = (x-1)^2 + 3$ so that the inverse is a function.

$x \geq 1$ or $x < 1$

Ex. 3) Determine the equation of the inverse for

a) $y = -2x + 1$

$x = -2y + 1$ switch x and y

$2y = 1 - x$ isolate y

$y = \frac{1-x}{2}$ $\rightarrow$ or $y = -\frac{1}{2}x + \frac{1}{2}$

$f^{-1}(x) = \frac{1-x}{2}$

inverse of an oblique line is another oblique line, which is a function ∴ use functional notation

b) $f(x) = -x^2 + 4$

$y = -x^2 + 4$

$x = -y^2 + 4$

$y^2 = 4 - x$

$y = \pm\sqrt{4-x}$ ← not a fcn

∴ do not use functional notation

Pre-Calculus 12 Enriched Transformations

Ex. 4) Determine whether or not $f(x) = -x^2 + 3, x \geq 0$ and $g(x) = \sqrt{3 - x}$ are inverses of each other.

In order to be inverses of each other, $f(g(x)) = x$ and $g(f(x)) = x$.

$f(x) = -x^2 + 3$

$f(g(x)) = -(\sqrt{3-x})^2 + 3$

$= -(3-x) + 3$

$= -3 + x + 3$

$f(g(x)) = x$

$g(x) = \sqrt{3-x}$

$g(f(x)) = \sqrt{3-(-x^2+3)}$

$= \sqrt{3+x^2-3}$

$= \sqrt{x^2}$

$g(f(x)) = x$

$\therefore$ they are inverses

pg 44

#1, 2b, c

3 b, c

4 a, c trig e

6

7 a, c