Pre-Calculus 12 Inverse Relations

An inverse relation is a relation that has been reflected in the line $y = x$.

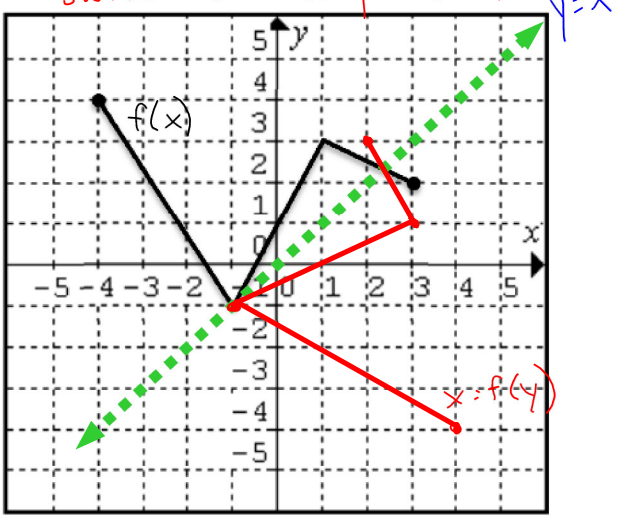
Notation:

- $x = f(y)$
- $y = f^{-1}(x)$

When the inverse is a function we use $f^{-1}(x)$ instead of $x = f(y)$

Ex. 1) Given the graph of $y = f(x)$

switch x and y values



a) Sketch the graph of its inverse

$y = f(x)$	inverse
$(-4, 4)$	$(4, -4)$
$(-1, -1)$	$(-1, -1)$
$(1, 3)$	$(3, 1)$
$(3, 2)$	$(2, 3)$

invariant point

b) Is the inverse a function? Explain.

No, fails the vertical line test

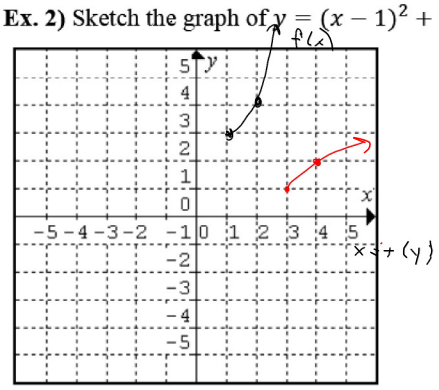
(more than one value of y for some values of x)

c) State the domain and range of the function and its inverse

	$f(x)$	inverse
D:	$[-4, 3]$	$[-1, 4]$
R:	$[-1, 4]$	$[-4, 3]$

The properties of x and y also switch

Ex. 2) Sketch the graph of $y = (x-1)^2 + 3$ and its inverse.



(2,4) → (4,2)
(1,3) → (3,1)
(0,4) → (4,0)

a) Is the inverse graph a function? No
does not pass v.l.t.

b) Determine two ways to restrict the domain of $y = (x-1)^2 + 3$ so that the inverse is a function.

$x \geq 1$ or $x \leq 1$

Ex. 3) Determine the equation of the inverse for:

a) $y = -2x + 1$

$x = -2y + 1$

$x - 1 = -2y$

$\frac{x-1}{-2} = y$

$f^{-1}(x) = \frac{x-1}{-2}$

same as $f^{-1}(x) = -\frac{1}{2}x + \frac{1}{2}$

① Switch x and y
② Solve/Isolate y
③ Use $f^{-1}(x)$ notation if it's a fcn

b) $y = -x^2 + 4$

$x = -y^2 + 4$

$y^2 = 4 - x$

$y = \pm\sqrt{4-x}$ ← not a fcn, don't use $f^{-1}(x)$

pg. 51
#1b, 2a, 4b, d
5b, e, 6, 13a, c, d

Review pg 56
Practice Test pg 58

Ex. 4) Determine whether or not $f(x) = -x^2 + 3, x \geq 0$ and $g(x) = \sqrt{3 - x}$ are inverses of each other.
In order to be inverses of each other, $f(g(x)) = x$ and $g(f(x)) = x$.

$f(x) = -x^2 + 3 \qquad x \geq 0$

$y = -x^2 + 3$

$x = -y^2 + 3$

$y^2 = 3 - x$

$y = +\sqrt{3 - x} \qquad y \geq 0$

$g(x)$

$\therefore$ inverses of each other