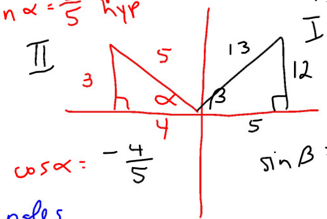
Pre-Calculus 12 Sum and Difference Identities…again

Ex. 1) If $\sin\alpha = \frac{3}{5}$ with α in QII and $\cos\beta = \frac{5}{13}$ with $\underbrace{\tan\beta > 0}^{QI}$ find the exact value of:

SOH CAH TOA

$\sin\alpha = \frac{3}{5} \frac{opp}{hyp}$ $\cos\beta = \frac{5}{13} \frac{adj}{hyp}$



$\cos\alpha = -\frac{4}{5}$ $\sin\beta = \frac{12}{13}$

3,4,5 triplet

5,12,13 triplet

$5^2 + y^2 = 13^2$

$y^2 = 169 - 25$

$y^2 = 144$

$y = \pm 12$

a) $\sin(\alpha+\beta)$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \underbrace{\frac{3}{5}\left(\frac{5}{13}\right)}_{\text{given these values}} + \underbrace{\left(-\frac{4}{5}\right)\left(\frac{12}{13}\right)}_{\text{use triangles to find these values}}$

$= \frac{15}{65} - \frac{48}{65}$

$\boxed{\sin(\alpha+\beta) = -\frac{33}{65}}$

b) $\cos(\alpha+\beta)$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(-\frac{4}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$

$= -\frac{20}{65} - \frac{36}{65}$

$\boxed{\cos(\alpha+\beta) = -\frac{56}{65}}$

c) $\tan(\alpha+\beta)$

$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$

$= \frac{-\frac{33}{\cancel{65}}}{-\frac{56}{\cancel{65}}}$

$\boxed{\tan(\alpha+\beta) = \frac{33}{56}}$

or use $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

d) The coordinates of $P(\alpha+\beta)$

$P(\alpha+\beta) = (\cos(\alpha+\beta), \sin(\alpha+\beta))$

$\left(-\frac{56}{65}, -\frac{33}{65}\right)$

in Quadrant III

Ex. 2) Expand $\boxed{\cos\left(\frac{\pi}{2}+\frac{\pi}{2}\right)}$ to verify that $\cos \pi = -1$.

$\pi = \frac{\pi}{2} + \frac{\pi}{2}$

$\cos\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \cos\frac{\pi}{2}\cos\frac{\pi}{2} - \sin\frac{\pi}{2}\sin\frac{\pi}{2}$

$= 0 \cdot 0 - 1 \cdot 1$

$\cos \pi = -1$

Ex. 3) Prove $\sin(\pi - x) = \sin x$

$\sin(\pi - x)$	$= \sin x$
$\sin\pi\cos x - \cos\pi\sin x$	
$\cancel{0 \cdot \cos x} - (-1)\sin x$	
$\sin x =$	RHS ✓

Ex. 4) Prove $\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

$\sin(A+B) + \sin(A-B)$	$= 2\sin A\cos B$
$\sin A\cos B + \cancel{\cos A\sin B} + \sin A\cos B - \cancel{\cos A\sin B}$	
$2\sin A\cos B =$	RHS ✓

Assignment: ~~Pg. 612: 6e, 8, 9a, 14b, 15a~~ pg. 306 # 5b, 7, 10, 11c, 20