MAAPC20S Linear Functions Lesson 4

Lesson Four – Slopes of Parallel/Perpendicular Lines

Parallel: lines do not intersect, same slope

// (symbol for parallel)

Example 1

Line GH passes through G(1, 3) and H(6, 10). Line JK passes through J(11, 10) and K(6, 3). Are the two lines parallel?

GH: $m = \frac{10-3}{6-1} = \frac{7}{5}$

JK: $m = \frac{3-10}{6-11} = \frac{-7}{-5} = \frac{7}{5}$

same slope

∴ the lines are parallel

Example 2

Line AB passes through A(1, 3) and B(5, 10). Line CD passes through C(0, –2) and D(11, 8). Are the two lines parallel?

$m_{AB} = \frac{10-3}{5-1} = \frac{7}{4}$ $\quad m_{CD} = \frac{8-(-2)}{11-0} = \frac{10}{11}$

different slopes

∴ lines are not parallel (//)

Example 3

Given the points R (–2, 0), S (6, 4), and T (–3, 4) determine the coordinates of point U on the *y*-axis so that TU is parallel to RS.

(0, y)

① Determine m_{RS} (same slope)

$m_{RS} = \frac{4-0}{6-(-2)} = \frac{4}{8} = \frac{1}{2}$

$\therefore m_{TU} = \frac{1}{2}$

②

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{1}{2} = \frac{y-4}{0-(-3)}$

$\frac{1}{2} = \frac{y-4}{3}$ cross mult

$1(3) = 2(y-4)$

$3 = 2y - 8$

$11 = 2y$

$\frac{11}{2} = y$

$\therefore U(0, \frac{11}{2})$

Slopes of Perpendicular Lines

meet at a right angle ⊥ (symbol for perpendicular)

- The slopes of two oblique perpendicular lines are negative reciprocals; that is, a line with slope a, $a \neq 0$, is perpendicular to a line with slope $-\frac{1}{a}$.

  ie 2 $\quad -\frac{1}{2}$

- If the slopes of two line segments are negative reciprocals, the line segments are perpendicular.

one line has a positive slope; the other has a negative slope

$\frac{3}{4}, -\frac{4}{3}$ ← negative reciprocals

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Example 1

perpendicular

If $\overline{AB} \perp \overline{DC}$ find the slope of $\overline{AB}$ given slope $\overline{DC}$:

a) $m\overline{DC} = -3$ $+\frac{1}{3}$

b) $m\overline{DC} = \frac{5}{4}$ $-\frac{4}{5}$

c) $m\overline{DC} = 0$ undefined

↑ horizontal, ⊥ line is vertical

or $m\overline{DC} = \frac{0}{1}$

$\therefore m\overline{AB} = \frac{-1}{0}$ ← undefined

Example 2

What is the slope of a line that is perpendicular to the line M(–5, 0) and N(0, 2).

x_1, y_1 x_2, y_2

① Calculate slope of MN first

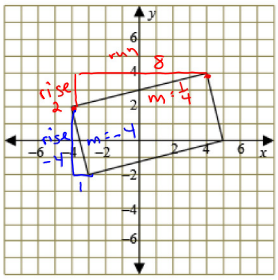
MN $m = \frac{2-0}{0-(-5)} = \frac{2}{5}$

$\therefore \perp m = -\frac{5}{2}$

↑ slope of line ⊥ to MN

Example 3

ABCD is a parallelogram. Is it a rectangle? Justify your answer.



Use graph to determine slope

Slopes are negative reciprocals of each other so the line segments are ⊥ (90° angles)

∴ it is a rectangle

Example 4

x_1, y_1 x_2, y_2

$\overline{AB}$ has coordinates A (1, –2), B (5, 4). $\overline{AC} \perp \overline{AB}$ and C lies on the x-axis. Find the coordinates of C.

(x, 0) x_2, y_2

y-coord is always 0 on the x-axis

① $m_{AB} = \frac{4-(-2)}{5-1} = \frac{6}{4} = \frac{3}{2}$

$\therefore \perp m = -\frac{2}{3}$

② $m = \frac{y_2 - y_1}{x_2 - x_1}$

$-\frac{2}{3} = \frac{0-(-2)}{x-1}$

$-\frac{2}{3} \times \frac{2}{x-1}$

$-2(x-1) = 3(2)$

$x - 1 = -3$

$x = -2$

$\therefore C(-2, 0)$

Steps

① Calculate slope of $\overline{AB}$

② Take negative reciprocal to get slope of $\overline{AC}$

③ Use ⊥m and coordinates to A and C in slope formula to determine the x-coordinate of C

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1a, c, e, g, i, k

2

3 a-c

4

6

10

13

m	$\perp m$	$\parallel m$
$\frac{2}{3}$	$-\frac{3}{2}$	$\frac{2}{3}$
$-\frac{7}{8}$	$\frac{8}{7}$	$-\frac{7}{8}$
undefined	0	undefined
5	$-\frac{1}{5}$	5
3	$-\frac{1}{3}$	3
$\frac{3}{4}$	$-\frac{4}{3}$	$\frac{3}{4}$