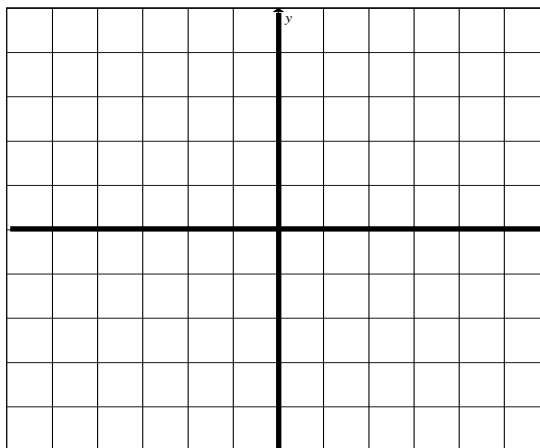# Lesson 4 Restrictions on Composite Functions
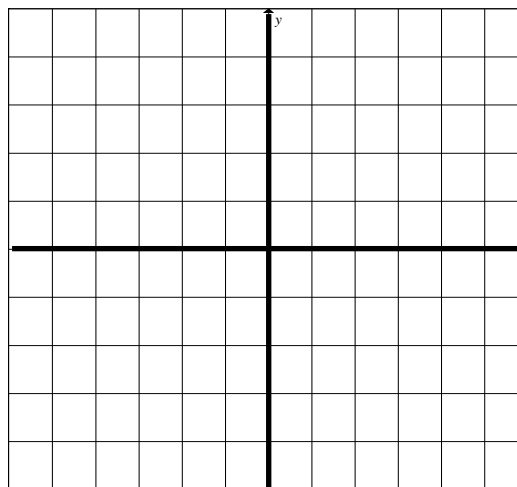
Examples

1. Given the functions $f(x) = x + 1$ and $g(x) = 4 - x^2$,

   a) state the domain and range of each function.

   b) sketch a graph of $y = f(g(x))$. State the domain and range of the new function.

   c) sketch a graph of $y = f(f(x))$. State the domain and range of the new function.

> The domain of the composite function, $f(g(x))$, is the set of all values of x in the domain of g such that $g(x)$ is in the domain of f.

Steps for finding the domain of composite functions without sketching a graph

1. Determine the domain of the inside functions. If there are any restrictions, keep them.

2. Determine the domain of the composite function.
   - If there are restrictions, add them to the restrictions from step 1.
   - If there is an overlap, use the more restrictive domain.

Examples

2. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 4$, determine an explicit equation for each of the following and then state the domain of the new function.

   a) $g(f(x))$

   b) $f(g(x))$

3. Determine possible functions f and g so that $y = f(g(x))$

   a) $y = (x - 2)^2$

   b) $y = \sqrt{x + 3}$

4. Given $f(x) = \frac{1}{x+3}$ and $g(x) = x^2 - 4x$, determine an explicit equation for each composite function below and state the domain of the new function.

   a) $g(f(x))$

   b) $f(g(x))$