L4 Fractional Exponents

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Lesson 4 Fractional Exponents and Radicals

Complete each table below. Use a calculator to complete the second column.

1 ^ (1 ÷ 2)
1 y^x (1 ÷ 2)

x	$x^{\frac{1}{2}} = \sqrt{x}$
1	1
4	2
9	3
16	4
25	5

x	$x^{\frac{1}{3}} = \sqrt[3]{x}$
1	1
8	2
27	3
64	4
125	5

Rational Number – A number that can be written in the form $\frac{m}{n}$ where m and n are integers $(n \neq 0)$.

To evaluate rational exponents we change from exponential form to radical form.

Square Root: $4^{\frac{1}{2}} = \sqrt{4} = 2$

Cube Root: $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

Rational Exponents: $a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{1}{n}}$ exists if n is an odd integer: $\sqrt[3]{8} = 2, \quad \sqrt[3]{-8} = -2$

$a^{\frac{1}{n}}$ exists if n is even, only if a is positive: $\sqrt{4} = 2, \quad \sqrt{-4} = \emptyset$

Examples: Evaluating Powers of the Form $a^{\frac{1}{n}}$

Evaluate each power without using a calculator

1. $27^{\frac{1}{3}}$ $\sqrt[3]{27} = 3$

2. $(-64)^{\frac{1}{3}}$ $\sqrt[3]{-64} = -4$

3. $\left(\frac{4}{9}\right)^{\frac{1}{2}}$ $\sqrt{\frac{4}{9}} = \frac{2}{3}$

Try
$81^{\frac{1}{2}}$ $\sqrt{81}$ 9
$(-125)^{\frac{1}{3}}$ $\sqrt[3]{-125}$ -5
$(-49)^{\frac{1}{2}}$ $\sqrt{-49}$ $\emptyset$

Extend $(a^m)^n$ to fractional exponents.

$4^{\frac{3}{2}} = (4^{\frac{1}{2}})^3 = (\sqrt{4})^3 = 2^3 = 8$

$a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m$ where $a \geq 0$ if n is even

Note: $\frac{m}{n}$ Must be in reduced form

Examples: Evaluating Powers of the Form $a^{\frac{m}{n}}$

4. Convert $\sqrt[4]{x^3}$ to exponential form.

$x^{\frac{3}{4}}$ $\quad \sqrt[5]{x^7} \quad x^{\frac{7}{5}}$

5. Convert $7^{\frac{2}{3}}$ to radical form.

$\sqrt[3]{7^2}$ $\quad 2^{\frac{3}{4}} \quad \sqrt[4]{2^3}$ or $(\sqrt[4]{2})^3$

6. Determine exact value without using the calculator:

a) $32^{\frac{2}{5}}$ $\quad (\sqrt[5]{32})^2 \quad 2^2 \quad 4$

Try $16^{\frac{3}{4}} \quad \sqrt[4]{16}^3 \quad 2^3 \quad 8$

b) $0.04^{\frac{3}{2}}$ $\quad \left(\frac{4}{100}\right)^{\frac{3}{2}} \quad \left(\frac{1}{25}\right)^{\frac{3}{2}}$ index $\quad \left(\sqrt{\frac{1}{25}}\right)^3 \quad \left(\frac{1}{5}\right)^3 \quad \frac{1}{125}$ (reduce)

c) $(-32)^{0.4}$ $\quad (-32)^{\frac{4}{10}} \quad (-32)^{\frac{2}{5}} \quad (\sqrt[5]{-32})^2 \quad (-2)^2 \quad 4$

d) $-27^{\frac{1}{3}}$ $\quad -\sqrt[3]{27} \quad -3$

e) $32^{\frac{3}{5}}$ $\quad (\sqrt[5]{32})^3 \quad 2^3 \quad 8$

f) $125^{\frac{2}{3}}$ $\quad (\sqrt[3]{125})^2 \quad 5^2 \quad 25$

Try $(-81)^{\frac{1}{2}} \quad -81^{\frac{1}{2}} \quad 25^{\frac{3}{2}} \quad (-100)^{\frac{3}{2}}$