Pre-Calculus 12 Sum and Difference Identities

Sum Formulas

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

Difference Formulas

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

We use these to find exact values not covered on the unit circle.

Ex. 1) Find the exact value of $\cos\frac{7\pi}{12}$

$$\frac{7\pi}{12} = \frac{4\pi}{12} + \frac{3\pi}{12} = \underbrace{\frac{\pi}{3}}_{\alpha} + \underbrace{\frac{\pi}{4}}_{\beta}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

* Watch notation

$$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$$ ← exact values from special circle

$$\cos\left(\frac{7\pi}{12}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$$ ← exact value

Ex. 2) Find the exact value of $\tan\frac{5\pi}{12}$

$$\frac{5\pi}{12} = \frac{2\pi}{12} + \frac{3\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$$

sum identity

or

$$\overset{\alpha}{\frac{8\pi}{12}} - \overset{\beta}{\frac{3\pi}{12}} = \frac{2\pi}{3} - \frac{\pi}{4}$$

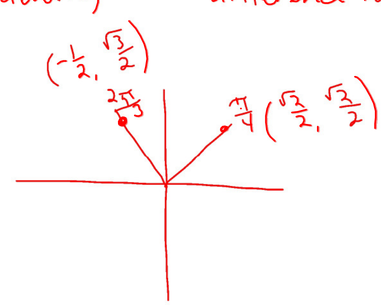
difference identity

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

$$\tan\left(\frac{2\pi}{3}-\frac{\pi}{4}\right) = \frac{\tan\frac{2\pi}{3} - \tan\frac{\pi}{4}}{1+\tan\frac{2\pi}{3}\tan\frac{\pi}{4}}$$

$$= \frac{-\sqrt{3}-1}{1+(-\sqrt{3})(1)}$$

$$= \frac{-\sqrt{3}-1}{1-\sqrt{3}}$$



Ex. 3) Simplify and find the exact value of: $\sin\underset{\alpha}{\frac{3\pi}{2}}\cos\underset{\beta}{\frac{5\pi}{4}} - \cos\underset{\alpha}{\frac{3\pi}{2}}\sin\underset{\beta}{\frac{5\pi}{4}}$

follows the pattern $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\sin\left(\frac{3\pi}{2}-\frac{5\pi}{4}\right) \quad \sin\frac{3\pi}{2}\cos\frac{5\pi}{4} - \cos\frac{3\pi}{2}\sin\frac{5\pi}{4}$

$$\therefore \sin\left(\frac{3\pi}{2}-\frac{5\pi}{4}\right)$$
$$\sin\left(\frac{6\pi}{4}-\frac{5\pi}{4}\right)$$
$$\sin\left(\frac{\pi}{4}\right)$$
$$\frac{\sqrt{2}}{2}$$

① Find the identity that is shown (right hand side)

② Determine combination on left hand side

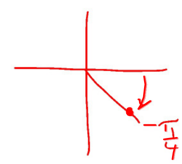
③ Add/subtract the angles

④ Determine trig fcn (exact value)

Ex. 4) Simplify and find the exact value of: $\cos\frac{\pi}{12}\cos\frac{\pi}{3} + \sin\frac{\pi}{12}\sin\frac{\pi}{3}$

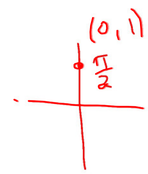
follows pattern of $\cos(\alpha-\beta)$

$$\cos\left(\frac{\pi}{12}-\frac{\pi}{3}\right)$$
$$\cos\left(\frac{\pi}{12}-\frac{4\pi}{12}\right)$$
$$\cos\left(-\frac{3\pi}{12}\right)$$
$$\cos\left(-\frac{\pi}{4}\right)$$
$$\frac{\sqrt{2}}{2}$$



Ex. 5) Express $\cos\left(\frac{\pi}{2}+x\right)$ as a function of x only.

$$\cos\left(\frac{\pi}{2}+x\right) = \cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x$$
$$= \cancel{0\cos x} \quad - 1\sin x$$
$$= -\sin x$$



Assignment: ~~Pg 612, #3b, 4c,d, 5b,e, 7a,c, MC# 1,2~~ pg 306 # 1b, d, 2a, d, 4c, 8c, e