Lesson 2 Regular Polygons

A polygon is a closed figure formed by joining three or more line segments at their endpoints. Polygons are classified according to the number of sides.

COMMON POLYGONS	
Name	*Number of sides*
triangle	3
quadrilateral	4
pentagon	5
hexagon	6
heptagon	7
octagon	8
nonagon	9
decagon	10
dodecagon	12

Regular polygon: a polygon with congruent side lengths and congruent interior angles (all sides are equal and all angles are equal).

Interior Angles

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Sum of Interior Angles

$S = 180^{\circ}(n - 2)$ where n is the # of sides of the regular polygon

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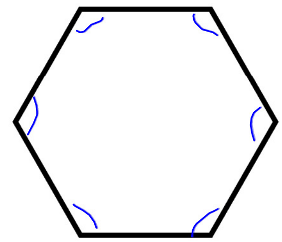
Size of Each Interior Angles

$M = \dfrac{180^{\circ}(n - 2)}{n}$ where M is the measure (size) of the angle

n is the # of sides of the regular polygon

Example 1

Determine the sum of the interior angles in a regular hexagon.



→ 6 sides

$\therefore n = 6$

$S = 180°(n-2)$

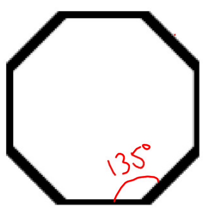
$S = 180°(6-2)$

$= 720°$ ← the 6 angles add up to 720°

Example 2

Determine the size of each interior angle in a regular octagon.

→ 8 sides, $\therefore n = 8$



$M = \frac{180°(n-2)}{n}$ ← sum of all angles split evenly between "n" angles

$M = \frac{180°(8-2)}{8}$

$= 135°$ ← size of each interior angle

Example 3

Calculate how many sides a regular polygon has if the sum of the interior angles is 1980°.

↖ S

$S = 180°(n-2)$

$\frac{1980°}{180°} = \frac{\cancel{180°}(n-2)}{\cancel{180°}}$

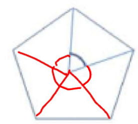
$11^{+2} = n \cancel{-2}^{+2}$

$\therefore n = 13$

13 sides

Central Angles

The central angle is the angle made at the centre of the polygon by any two adjacent vertices of the polygon.
All central angles will add up to 360°.



on formula sheet

Central angle $= \frac{360°}{n}$ **where *n* is the number of sides of a polygon**

Example 1

Determine the measure of the central angle of a decagon.

↳ 10 sides, ∴ $n = 10$

$$C = \frac{360°}{10}$$

$$= 36°$$

Example 2

Determine the number of sides of a regular polygon with a central angle of 30°.

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$$n = \frac{360°}{\text{central angle}}$$

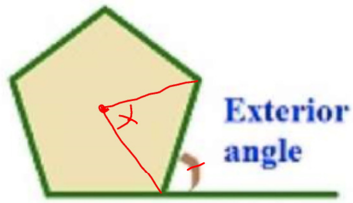
$$n = \frac{360°}{30°}$$

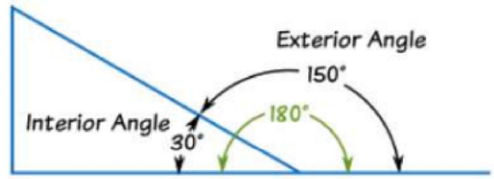
$$n = 12$$

∴ 12 sides (dodecagon)

Exterior Angles

In a regular polygon, the measure of the exterior angle is equal to the measure of the central angle. Therefore, the sum of the exterior angles is also 360°.
The interior and exterior angles add up to 180°. supplementary





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Size of exterior angle = 180° - interior angle

Number of sides of a polygon = $\dfrac{\mathbf{360°}}{\textbf{Exterior Angle}}$

Example 1

A regular polygon has equal exterior angles of 72°.

a.) Determine the size of each interior angle in this regular polygon.

Interior angle $= 180° - 72°$
$= 108°$

b.) Determine the number of sides in this regular polygon.

$n = \dfrac{360°}{\text{exterior angle}}$

$n = \dfrac{360°}{72°}$

$= 5$ sides (pentagon)

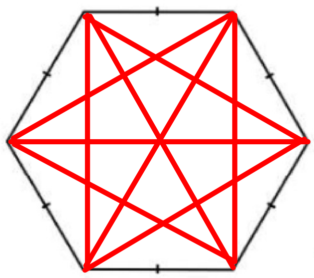
Diagonals of a Regular Polygon

on formula sheet

$\mathbf{D} = \frac{n(n-3)}{2}$ **where n is the number of sides of a polygon**

Example 1

Given the following regular polygon:



9 diagonals

6 sides

$n = 6$

Calculate or illustrate the total number of diagonals that can be drawn.
If illustrating, clearly state the total number of diagonals.

$$D = \frac{n(n-3)}{2}$$

$$D = \frac{6(6-3)}{2}$$

$$= 9 \text{ diagonals}$$