

Pre-Calculus 12 Calculating Circular Functions

Equation of the unit circle: $x^2 + y^2 = 1$
 $x^2 + y^2 = r^2$

- Represents the circle with centre at the origin and radius 1 unit.

pt $(\frac{3}{5}, \frac{4}{5})$

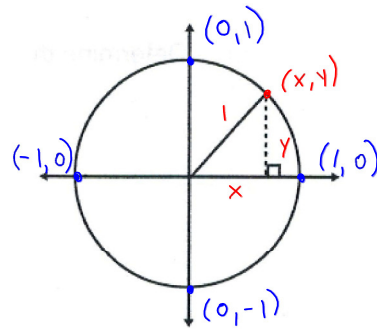
This point is on the unit circle

$$x^2 + y^2 = 1$$

$$(\frac{3}{5})^2 + (\frac{4}{5})^2 = 1$$

$$\frac{9}{25} + \frac{16}{25} = 1$$

$$\frac{25}{25} = 1 \checkmark$$

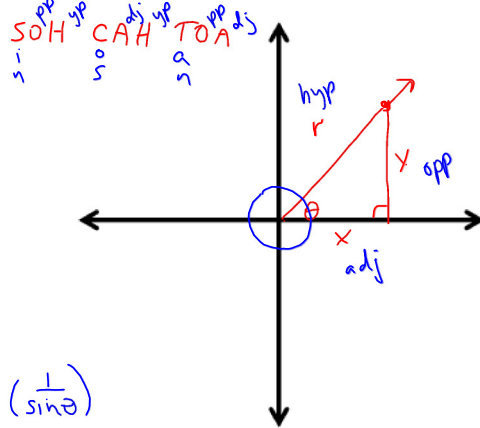


For any angle, θ in standard position, with terminal point $P(x, y)$ on a circle with radius, r :

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x} \text{ or } \frac{\sin \theta}{\cos \theta}$$



Reciprocal trig fns

(cosecant) $\csc \theta = \frac{r}{y}$ ← reciprocal of $\sin \theta$ ($\frac{1}{\sin \theta}$)

(secant) $\sec \theta = \frac{r}{x}$ ← reciprocal of $\cos \theta$ ($\frac{1}{\cos \theta}$)

(cotangent) $\cot \theta = \frac{x}{y}$ ← reciprocal of $\tan \theta$ ($\frac{1}{\tan \theta}$)
 or $\frac{\cos \theta}{\sin \theta}$

Ex. 1) Determine the value to the nearest thousandth

(3 decimal places)

a) $\sin 106^\circ = 0.961$

b) $\csc 64^\circ = \frac{1}{\sin 64^\circ} = 1.113$

c) $\cot(-88^\circ) = \frac{1}{\tan(-88^\circ)} = -0.035$

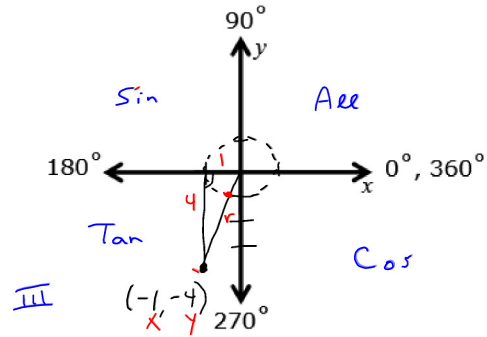
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Ex. 2) P(-1, -4) is a terminal point of angle θ in standard position. Determine the exact values of the six trigonometric ratios.

$$\begin{aligned}
 x^2 + y^2 &= r^2 \\
 1^2 + 4^2 &= r^2 \\
 17 &= r^2 \\
 \pm\sqrt{17} &= r \\
 \text{radius is always +ve so } r &= \sqrt{17} \\
 \text{reciprocal} \\
 \csc \theta &= -\frac{\sqrt{17}}{4} \\
 \sec \theta &= -\sqrt{17} \\
 \cot \theta &= \frac{1}{4}
 \end{aligned}$$

radius is always +ve so

$$\begin{aligned}
 \sin \theta &= \frac{y}{r} \\
 &= \frac{-4}{\sqrt{17}} \\
 \cos \theta &= \frac{x}{r} \\
 &= \frac{-1}{\sqrt{17}} \\
 \tan \theta &= \frac{y}{x} \\
 &= \frac{-4}{-1} = 4
 \end{aligned}$$

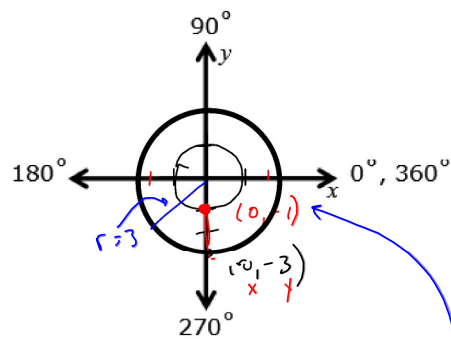


$$\begin{aligned}
 -1 &\leq \sin \theta \leq 1 \\
 -1 &\leq \cos \theta \leq 1
 \end{aligned}$$

Ex. 3) Given P(0, -3) is a terminal point of an angle in standard position, determine the exact values of the 6 trigonometric ratios.

$$\begin{aligned}
 \sin \theta &= \frac{y}{r} \\
 &= \frac{-3}{3} \\
 &= -1 \\
 \csc \theta &= -1 \\
 \cos \theta &= \frac{x}{r} \\
 &= \frac{0}{3} \\
 &= 0 \\
 \sec \theta &= \frac{3}{0} \\
 &= \text{undefined} \\
 \tan \theta &= \frac{y}{x} \\
 &= \frac{-3}{0} \\
 &= \text{undefined} \\
 \cot \theta &= \frac{0}{-3} \\
 &= 0
 \end{aligned}$$

(0, -3) r=3



(x, y)
on the unit circle
is
(cos θ , sin θ)

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Ex. 4) Suppose $\sec\theta = 4$

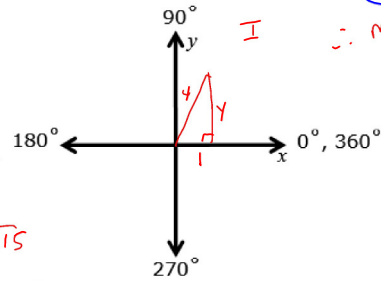
a) Determine the exact values of the other trigonometric ratios for $0^\circ \leq \theta \leq 180^\circ$

reciprocal $\cos\theta = \frac{1}{4} \quad \frac{x}{r}$

$\sin\theta = \frac{\sqrt{15}}{4} \quad \csc\theta = \frac{4}{\sqrt{15}}$

$\tan\theta = \frac{\sqrt{15}}{1} \quad \cot\theta = \frac{1}{\sqrt{15}}$
 $= \sqrt{15}$

$x^2 + y^2 = r^2$
 $1^2 + y^2 = 4^2$
 $y^2 = 15$
 $y = \pm\sqrt{15}$
 in QI $\therefore y = \sqrt{15}$



QI or II
 $\cos\theta > 0$
 QI or IV
 \therefore must be in QI

b) Determine the possible values of θ in the domain of $-360^\circ \leq \theta \leq 360^\circ$

$\cos\theta = \frac{1}{4}$

$\theta = \cos^{-1}\left(\frac{1}{4}\right)$

$\theta_r = 75.522^\circ$

$\cos\theta > 0$ in QI, IV

QI $\theta = \theta_r = 75.522^\circ$

QIV $\theta = 360^\circ - \theta_r = 360^\circ - 75.522^\circ = 284.478^\circ$

find coterminal angles $-360^\circ \leq \theta \leq 360^\circ$

$\theta_c = 75.522^\circ - 360^\circ = -284.478^\circ$

$\theta_c = 284.478^\circ - 360^\circ = -75.522^\circ$

$\therefore \theta = -284.478^\circ, -75.522^\circ, 75.522^\circ, 284.478^\circ$

Assignment: Pg. 47 #3, Pg. 477: 8, 9, 10, 11 Pg. 186 # 2 a, e 3 c, Pg. 202 # 5 b, d, 13, 19 b

Worksheet

1a) $\sin\theta > 0, \sec\theta > 0$
 $\cos\theta > 0$

QI

| | |
|-----------|-----------|
| II ✓ S | QI ✓ A |
| III T | IV ✓ C |

find trig ratios