Pre-Calculus 12
Reciprocal and Quotient Identities

A trigonometric identity is an equation involving trig functions that is true for all permissible values of the variable(s).

Identities can be verified:

- numerically (by substituting specific values for the variable)
- graphically (using technology)

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta},\ \sin\theta \neq 0$

$\sec\theta = \frac{1}{\cos\theta},\ \cos\theta \neq 0$

$\cot\theta = \dfrac{1}{\tan\theta}, \sin\theta \neq 0\ and\ \cos\theta \neq 0$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}, \cos\theta \neq 0$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}, \sin\theta \neq 0$

Proving Identities

To prove that an identity is true for all permissible values, it is necessary to express both sides of the identity in equivalent forms. One or both sides of the identity must be algebraically manipulated into an equivalent form to match the other side.

You **CANNOT** perform operations across the equal sign when proving a potential identity. Simplify the expressions on each side of the identity independently.

Verifying an Identity - using a specific value validates that it is true for that value only.

Proving an Identity - is done algebraically and validates the identity for all permissible values of the variable.

Ex. 1) For the identity $(\sec\theta)(1+\cos\theta) = 1 + \sec\theta$:

a) verify the identity for $\theta = 30°$

b) prove the identity

a) $(\sec\theta)(1+\cos\theta) = 1+\sec\theta$

LHS	RHS
$(\sec 30°)(1+\cos 30°)$	$1+\sec 30°$
$\frac{2}{\sqrt{3}}\left(1+\frac{\sqrt{3}}{2}\right)$	$1+\frac{2}{\sqrt{3}}$
$\frac{2}{\sqrt{3}}+1$	$1+\frac{2}{\sqrt{3}}$

LHS = RHS ✓

b) $(\sec\theta)(1+\cos\theta) = 1+\sec\theta$

LHS	RHS
$\sec\theta\left(1+\frac{1}{\sec\theta}\right)$	
$\sec\theta + 1$	= RHS ✓

Ex. 2) For the identity $1 - \tan\theta = \frac{\cot\theta - 1}{\cot\theta}$:

a) Verify the identity for $\theta = \frac{\pi}{4}$.

b) Prove the identity

$\tan\frac{\pi}{4}$

$\frac{\pi}{4}\ \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

a) $1 - \tan\theta = \frac{\cot\theta - 1}{\cot\theta}$

LHS	RHS
$1-\tan\frac{\pi}{4}$	$\frac{\cot\frac{\pi}{4}-1}{\cot\frac{\pi}{4}}$
$1-1$	$\frac{1-1}{1}$
0	0

LHS = RHS ✓

$\frac{1+3}{4}$

$\frac{1}{4}+\frac{3}{4}$

b) $1-\tan\theta = \frac{\cot\theta - 1}{\cot\theta}$

LHS	RHS
	$\frac{\cot\theta}{\cot\theta} - \frac{1}{\cot\theta}$
LHS =	$1 - \tan\theta$ ✓

Ex. 3) For the identity $\frac{\cot\theta}{\csc\theta} = \cos\theta$:

a.) Determine the non-permissible values of θ ← general solution (radians)

b.) Prove the identity

a) npv's denom is 0

$\csc\theta = 0$

$\frac{1}{\sin\theta} = 0$

$\sin\theta = 0$ when

$\theta = k\pi, \; k \in \mathbb{Z}$

$\cot\theta$

$\frac{\cos\theta}{\sin\theta}$

b) $\frac{\cot\theta}{\csc\theta} = \cos\theta$

$\frac{\frac{\cos\theta}{\cancel{\sin\theta}}}{\frac{1}{\cancel{\sin\theta}}}$

$\cos\theta = RHS$ ✓

$\frac{\cos\theta}{\sin\theta} \div \frac{1}{\sin\theta}$

$\frac{\cos\theta}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{1}$

$\frac{\pi}{2}$, $\frac{3\pi}{2}$

Ex. 4) For the identity $\boxed{\frac{\sin\theta + \tan\theta}{1+\sec\theta} = \sin\theta}$:

a.) determine the non-permissible values of θ — quotient identities, reciprocal identities, denominator } values which make any of these equal to 0

b.) prove the identity

a) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$

$\cos\theta = 0$

$\theta = \frac{\pi}{2} + k\pi, \; k \in \mathbb{Z}$

denom = 0

$1 + \sec\theta = 0$

$\sec\theta = -1$

$\frac{1}{\cos\theta} = -1$

$\cos\theta = -1$ ← reciprocal of both sides

$\theta = \pi + 2k\pi, \; k \in \mathbb{Z}$

b) $\frac{\sin\theta + \tan\theta}{1+\sec\theta} = \sin\theta$

$\frac{\left(\frac{\cos\theta}{\cos\theta}\right)\frac{\sin\theta}{1} + \frac{\sin\theta}{\cos\theta}}{\left(\frac{\cos\theta}{\cos\theta}\right)1 + \frac{1}{\cos\theta}}$

LCD $\cos\theta$

$\frac{\frac{\sin\theta\cos\theta + \sin\theta}{\cancel{\cos\theta}}}{\frac{\cos\theta + 1}{\cancel{\cos\theta}}}$

$\frac{\sin\theta\cos\theta + \sin\theta}{\cos\theta + 1}$

GCF $\sin\theta$

$\frac{\sin\theta\,(\cancel{\cos\theta + 1})}{\cancel{\cos\theta + 1}}$

$\sin\theta = RHS$ ✓

Ex. 5) Prove the identity:

$$\frac{\tan\theta csc^2\theta}{sec^2\theta} = \cot\theta$$

$$\frac{\cancel{\sin}\theta}{\cos\theta} \cdot \frac{1}{\sin^{\cancel{2}}\theta}$$

$$\frac{1}{\cos^2\theta}$$

$$\frac{\frac{1}{\cos\theta\sin\theta}}{\frac{1}{\cos^2\theta}}$$

$$\frac{1}{\cancel{\cos\theta}\sin\theta} \cdot \cos^{\cancel{2}}\theta$$

$$\frac{\cos\theta}{\sin\theta}$$

$\cot\theta$ = RHS ✓

pg. 296 # 1b, c
5a, b
6a, c

worksheet
5, 8, 9, 12,
16

Assignment: Pg. 611; #3b,d, 4c,d, 5b, 7b, 8a, 9b, 11