Lesson 3 Introduction to Logarithms

Recall: The inverse of the exponential function $y = a^x$ is $x = a^y$. This inverse is also called a logarithmic function and is written as $y = log_a x$ (***Read as***: "y equals the log of x in base a"), where "a" is a positive number other than 1.

Log Form ⟷ **Exponential Form**

$log_a x = y$ (argument, base, exponent)

$a^y = x$ (exponent, base, argument goes here)

Common log – a log with base 10 (our number system is based on powers of 10)

ie) $\log_{10} 25$ ← implied to be a base of 10

Ex. 1) Express the following in logarithmic form

a) $2^3 = 8$ (exp, base, argument) → $\log_2 8 = 3$ (2 stays as the base; 3 ⇄ 8 switch sides)

b) $3^{-2} = \frac{1}{9}$ → $\log_3 \frac{1}{9} = -2$

c) $y = 4^x$ → $\log_4 y = x$

d) $A^2 = C$ → $\log_A C = 2$

Ex. 2) Express the following in exponential form

a) $\log_4 16 = 2$ (argument, exp, base) → $4^2 = 16$ (4 stays as base; 16 ⇄ 2 switch sides)

b) $\log \frac{1}{1000} = -3$ → $10^{-3} = \frac{1}{1000}$

c) $y = \log_{\frac{1}{2}} 4$ → $\left(\frac{1}{2}\right)^y = 4$

d) $M = \log_b N$ → $b^M = N$

Ex. 3) Evaluate

a) $\log_2 16$

4

Let $\log_2 16 = x$

$2^x = 16$

$x = 4$

b) $\log_2\left(\frac{1}{4}\right)$

-2

$\log_2 16 = 4$ makes sense because in exp form $2^4 = 16$

$\log_2\left(\frac{1}{4}\right) = -2$

c) $\log_3(\sqrt{3})$

$\frac{1}{2}$

$3^{\frac{1}{2}}$

$\log_3 3^{\frac{1}{2}} = \frac{1}{2}$

$3^{\frac{1}{2}} = 3^{\frac{1}{2}}$

d) $\log_3(\log_2 8)$

evaluate first

$\log_2 8 = 3$

*Think 2 to what power gives me 8

$\log_3 3$

1

Ex. 4) Solve

a.) $\log_8 x = \frac{1}{3}$

Write in exp form

$8^{\frac{1}{3}} = x$

Solve

$\sqrt[3]{8} = x$

$2 = x$

b) $\log_8 64 = y$

$8^y = 64$

$y = 2$

Ex. 5) Without technology, estimate the value of $\log_2 14$ to one decimal place.

Let $\log_2 14 = x$

$2^x = 14$ ← exp form

2^3 | 2^x | 2^4 ← Place between benchmarks

8 | $\log_2 14$ | 16

$\log_2 8$ | 3.7 | $\log_2 16$

3 | | 4

$\log_2 14 \approx 3.7$

pg 167

1, 2, 3

a, c, e, g, i

4 a, c, e, h, k, l,

n, p, r, s, t