$y = a^x$
$x = a^y$

reflection over the line $y = x$

Pre-Calculus 12 Introduction to Logarithms

For the exponential function $y = a^x$ the inverse is $x = a^y$. This inverse is also a function and is called a logarithmic function. It is written as $y = log_a x$ (***Read as***: "y equals the log of x in base a"), where "a" is a positive number other than 1.

base, argument

Log Form	⟷	Exponential Form
$log_a x = y$		$a^y = x$

argument, exponent, base, exponent, base, argument

Common log – a log with base 10 (our number system is based on powers of 10)

ie) log 25 same as $log_{10} 25$

10 is implied so does not need to be included

Ex. 1) Express the following in logarithmic form

a) $2^3 = 8$ switch $\log_2 8 = 3$ (base, exp)

2 stays as the base
3 & 8 switch sides

b) $3^{-2} = \frac{1}{9}$ $\log_3 \left(\frac{1}{9}\right) = -2$ (base, exponent)

① Place the base
② Switch sides

c) $y = 4^x$ $\log_4 y = x$

d) $A^2 = C$ $\log_A C = 2$ ← always equal to the exponent

Ex. 2) Express the following in exponential form

a) $\log_4 16 = 2$ (base, exponent) $4^2 = 16$

b) $\log \frac{1}{1000} = -3$ base implied as common base 10 $10^{-3} = \frac{1}{1000}$

Place the base
Switch sides

c) $y = \log_{\frac{1}{2}} 4$ $\left(\frac{1}{2}\right)^y = 4$

d) $M = \log_b N$ $b^M = N$

Ex. 3) Evaluate

a) $\log_2 16$

4

since $2^4 = 16$

makes sense in exp form

"what power of 2 equals 16"

$\log_2 16$ = exponent

b) $\log_2\left(\frac{1}{4}\right)$

-2

since $2^{-2} = \frac{1}{4}$

Let $x = \log_2\left(\frac{1}{4}\right)$

$2^x = \frac{1}{4}$

$2^{-2} = \frac{1}{4}$

c) $\log_3(\sqrt{3})$

$\frac{1}{2}$

recall $\sqrt{3} = 3^{\frac{1}{2}}$

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

d) $\log_3(\log_2 8)$

evaluate first

$\log_2 8 = 3$

$\log_3 3$

1

$2^? = 8$

* think 2 to what power equals 8

Ex. 4) Solve

a.) $\log_8 x = \frac{1}{3}$

exp form $8^{\frac{1}{3}} = x$

or $\sqrt[3]{8} = x$

$2 = x$

b) $\log_8 64 = y$

$2 = y$

or exp form $8^y = 64$

$\therefore y = 2$

Ex. 5) Without technology, estimate the value of $\log_2 14$ to one decimal place.

Place between benchmarks

$2^3 = 8$ 2^x $2^4 = 16$

$3 < \log_2 14 < 4$

≈ 3.7 or 3.8

$\log_2 14 \approx 3.7$

$2^{3.7} \approx 14$

pg. 380 # 2, 3, 4, 12, 14