

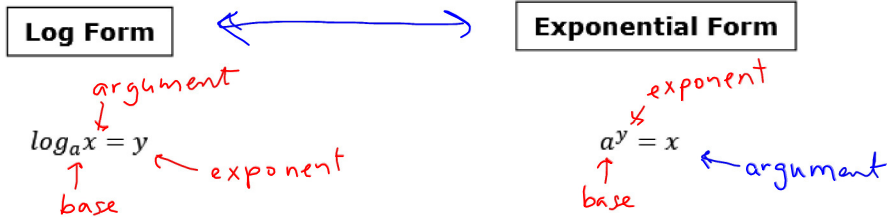
$$y = a^x$$

$$x = a^y$$

reflection over the line $y = x$

Pre-Calculus 12 Introduction to Logarithms

For the exponential function $y = a^x$ the inverse is $x = a^y$. This inverse is also a function and is called a logarithmic function. It is written as $y = \log_a x$ (Read as: "y equals the log of x in base a"), where "a" is a positive number other than 1.



Common log – a log with base 10 (our number system is based on powers of 10)

ie) $\log 25$

same as $\log_{10} 25$

10 is implied so does not need to be included

Ex. 1) Express the following in logarithmic form

2 stays as the base
3 & 8 switch sides

a) $2^3 = 8$ switch

$$\log_2 8 = 3$$

↑ base ← exp

b) $3^{-2} = \frac{1}{9}$

① Place the base
② Switch sides

$$\log_3 \left(\frac{1}{9}\right) = -2$$

↑ base ← exponent

c) $y = 4^x$

$$\log_4 y = x$$

d) $A^2 = C$

$$\log_A C = 2$$

← always equal to the exponent

Ex. 2) Express the following in exponential form

a) $\log_4 16 = 2$

↑ base ← exponent

$$4^2 = 16$$

Place the base
switch sides

c) $y = \log_{\frac{1}{2}} 4$

$$\left(\frac{1}{2}\right)^y = 4$$

b) $\log_{1000} \frac{1}{1000} = -3$

base implied as common base 10

$$10^{-3} = \frac{1}{1000}$$

d) $M = \log_b N$

$$b^M = N$$

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Ex. 3) Evaluate $\log_2 16 = \text{exponent}$

a) $\log_2 16$ "what power of 2 equals 16" b) $\log_2 \left(\frac{1}{4}\right)$

4 since $2^4 = 16$ -2 since $2^{-2} = \frac{1}{4}$

makes sense in exp form

Let $x = \log_2 \left(\frac{1}{4}\right)$

$$2^x = \frac{1}{4}$$

$$2^{-2} = \frac{1}{4}$$

c) $\log_3(\sqrt{3})$ $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$\frac{1}{2}$ recall $\sqrt{3} = 3^{\frac{1}{2}}$

d) $\log_3(\log_2 8)$ evaluate first

$\log_3 3$ $\log_2 8 = 3$

1 $2^3 = 8$

* Think 2 to what power equals 8

Ex. 4) Solve

a) $\log_8 x = \frac{1}{3}$

exp form $8^{\frac{1}{3}} = x$

or $\sqrt[3]{8} = x$

$2 = x$

b) $\log_8 64 = y$

$2 = y$ or $8^y = 64$

$\therefore y = 2$

Ex. 5) Without technology, estimate the value of $\log_2 14$ to one decimal place.

Place between benchmarks

$$2^3 = 8 \quad 2^x \quad 2^4 = 16$$

$$3 < \log_2 14 < 4$$

$$\approx 3.7 \text{ or } 3.8$$

$\log_2 14 = 3.7$

$2^{3.7} \approx 14$

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2, 3, 4, 12,
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