Integration by Parts (cont'd)

ex.1 Find $\int x^2 e^x \, dx$

$u = x^2 \qquad dv = e^x dx$

$du = 2x\,dx \qquad v = e^x$

$$\int u\,v - \int v\,du$$

$$\int x^2 e^x \, dx = x^2 e^x - \int e^x \cdot 2x \, dx$$

$$= x^2 e^x - 2\int x e^x \, dx$$

Integration by parts again!

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = e^x$

$$= x^2 e^x - 2\left(xe^x - \int e^x dx\right)$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

ex.2 Find $\int e^x \cos x \, dx$

$u = \cos x \qquad dv = e^x dx$

$du = -\sin x\,dx \qquad v = e^x$

$$\int e^x \cos x \, dx = \cos x\, e^x - \int e^x(-\sin x\, dx)$$

$$= \cos x\, e^x + \int e^x \sin x \, dx$$

$u = \sin x \qquad dv = e^x dx$

$du = \cos x\,dx \qquad v = e^x$

$$\int e^x \cos x\,dx = \cos x\, e^x + e^x \sin x - \int e^x \cos x\, dx$$

same as left hand side

$$2\int e^x \cos x\,dx = e^x \cos x + e^x \sin x$$

$$\int e^x \cos x\,dx = \frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + C$$

$$\text{or} \quad = \frac{1}{2} e^x (\cos x + \sin x) + C$$

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Ans

8) $2x^2 \sin\left(\frac{x}{2}\right) + 8x\cos\left(\frac{x}{2}\right) - 16\sin\left(\frac{x}{2}\right) + C$

10) $\frac{t^3}{3}\ln t - \frac{t^3}{9} + C$