Integration By Parts

Formula

$$\boxed{\int u\,dv = u \cdot v - \int v\,du}$$

* Used when integral contains 2 different types of fcns where one is not the derivative of the other.

To pick "u"

- Logs
- Inv trig
- Algebraic
- Trig
- Exp

Follow this order to choose which fcn is "u"

ex. 1 Evaluate

$$\int x \sin(3x)\,dx$$

$u = x$ ← Algebraic $dv = \sin(3x)\,dx$ ← Trig

$du = dx$ $v = -\frac{1}{3}\cos(3x)$

$$\int x\sin(3x)\,dx = \overset{u}{x}\Big(\overset{v}{-\tfrac{1}{3}\cos(3x)}\Big) - \int \overset{v}{-\tfrac{1}{3}\cos(3x)}\,\overset{du}{dx}$$

$$= -\frac{1}{3}x\cos(3x) + \frac{1}{3}\left(\frac{1}{3}\sin(3x)\right) + C$$

$$= -\frac{1}{3}x\cos(3x) + \frac{1}{9}\sin(3x) + C$$

$u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$\int \sin(3x)\,dx$

$\frac{1}{3}\int \sin u\,du$

$-\frac{1}{3}\cos u$

ex. 2 Find $\int x e^{2x}\,dx$

$u = x$ ← Alg $dv = e^{2x}\,dx$ ← Exp

$du = dx$ $v = \frac{1}{2}e^{2x}$

$$\int x e^{2x}\,dx = x\left(\frac{1}{2}e^{2x}\right) - \int \frac{1}{2}e^{2x}\,dx$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{2}\left(\frac{1}{2}e^{2x}\right) + C$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$$

ex. 3 Find $\int \ln x\,dx$

$u = \ln x$ $dv = dx$

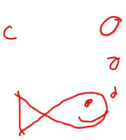
$du = \frac{1}{x}\,dx$ $v = x$

$$\int \ln x\,dx = x\ln x - \int x\left(\frac{1}{x}\,dx\right)$$

$$= x\ln x - \int dx$$

$$= x\ln x - x + C$$

$$= x(\ln x - 1) + C$$



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1, 3, 4, 5, 9

#4 ans

$\frac{2}{3}t\sin(3t) + \frac{2}{9}\cos(3t) + C$