Pre-Calculus 11 Infinite Geometric Series

An ***infinite geometric series*** has an infinite number of terms:

$16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \ldots$ ← represents infinite # of terms

If the sequence ***converges*** to a constant value as the number of terms increases, then the geometric series is convergent and the constant value is the finite sum of the series.

This sum is called the sum to infinity and is denoted by S_∞

For an infinite geometric series with first term, t_1, and common ratio, $-1 < r < 1$, the sum of the series, S_∞, is:

$$S_\infty = \frac{t_1}{1-r}$$

Examples

1. Determine whether each infinite geometric series converges or diverges.

a) $\frac{1}{3} + \frac{1}{12} + \frac{1}{48} + \frac{1}{192} + \ldots$

$-1 < \frac{1}{4} < 1$

$r = \frac{1}{4}$ $\therefore$ Series is convergent

b) $-4 - 8 - 16 - 32 - \cdots$

$r = 2$ $\therefore$ series is divergent

c) $\frac{1}{10} - \frac{1}{100} + \frac{1}{1000} - \frac{1}{10\,000} + \ldots$

$r = -\frac{1}{10}$ $-1 < -\frac{1}{10} < 1$

$\therefore$ series is convergent

$$16+8+4+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\frac{1}{128}+\frac{1}{256}$$

$0.5 + 0.25 + 0.125 + 00625 + 0.03125 + 0.015625 + 0.0078125$

$+ 0.00390625 + 0.001953125 + 0.0009765625$

$S_2 = 24$
$S_3 = 28$
$S_4 = 30$
$S_5 = 31$
$S_6 = 31.5$

$S_7 = 31.75$
$S_8 = 31.875$
$S_9 = 31.8815$
$S_{10} = 31.9125$
$S_{11} = 31.928125$
$S_{12} = 31.9359375$
$S_{13} = 31.93984375$
$S_{14} = 31.941796875$
$S_{15} = 31.9427734375$

$$S_\infty = \frac{16}{1-\frac{1}{2}}$$
$$= 32$$

2. Determine the sum of each geometric series, if it exists.

a) $32 + 8 + 2 + \frac{1}{2} + \ldots$

$$S_\infty = \frac{t_1}{1-r} = \frac{32}{1-\frac{1}{4}} = \frac{32}{\frac{3}{4}}$$

$$S_\infty = 32 \cdot \frac{4}{3} = \frac{128}{3}$$

or 42.67

b) $100 - 10 + 1 - \frac{1}{10} + \ldots$

$$S_\infty = \frac{t_1}{1-r} = \frac{100}{1-(-\frac{1}{10})} = \frac{100}{\frac{11}{10}} = \frac{1000}{11}$$

or 90.91

c) $1 + 8 + 64 + \ldots$

$r = 8$

$\therefore$ series is divergent

an infinite sum does not exist

3. The sum of an infinite geometric series is 16. If the common ratio is $\frac{1}{2}$, find the value of t_1.

$$S_\infty = \frac{t_1}{1-r}$$

$$16 = \frac{t_1}{1-\frac{1}{2}}$$

$$16 = \frac{t_1}{\frac{1}{2}}$$

$$8 = t_1$$

Assignment: Pg. 68; #3a,b, 4a,c, 5b,c, 7, 8a, 10, MC#1-3