Implicit Differentiation

- useful for finding derivatives when y is not defined explicitly in terms of x.

ex.1 Find $\frac{dy}{dx}$ for the relation $x^2 + y^2 = 16$

Differentiate both sides of the eqn w.r.t. x

$$x^2 + y^2 = 16$$

$$2x\frac{dx}{dx} + 2y\frac{dy}{dx} = 0$$

isolate $\frac{dy}{dx}$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

ex.2 Find the derivative, using implicit differentiation

$$y^4 + 3y - 4x^3 = 5x + 1$$

$$4y^3\frac{dy}{dx} + 3\frac{dy}{dx} - 12x^2 = 5$$

$$4y^3\frac{dy}{dx} + 3\frac{dy}{dx} = 5 + 12x^2$$

Factor $\frac{dy}{dx}$

$$\frac{dy}{dx}(4y^3 + 3) = 12x^2 + 5$$

$$\frac{dy}{dx} = \frac{12x^2 + 5}{4y^3 + 3}$$

ex.3 Determine $\frac{dy}{dx}$ for the relation $2x^2 - xy - 2y^2 = 1$

$$2x^2 - xy - 2y^2 = 1$$

$$4x - \left(x(1)\frac{dy}{dx} + (1)y\right) - 4y\frac{dy}{dx} = 0$$

product rule

$$4x - x\frac{dy}{dx} - y - 4y\frac{dy}{dx} = 0$$

$$4x - y = x\frac{dy}{dx} + 4y\frac{dy}{dx}$$

$$4x - y = \frac{dy}{dx}(x + 4y)$$

$$\frac{4x - y}{x + 4y} = \frac{dy}{dx}$$

where $x + 4y \neq 0$

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