Pre-Calculus 11
Graphing Reciprocals of Quadratic Functions

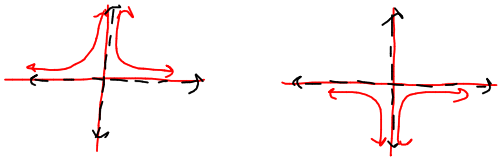
When we graph the reciprocal of a quadratic function, the quadratic function may have 0, 1, or 2 vertical asymptotes. (parabolas have 0, 1 or 2 x-ints)

There are 3 basic shapes

Shape 1 – Funnel or Inverted Funnel

- This shape has one vertical asymptote

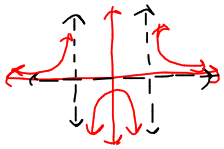
$y = \frac{1}{x^2}$ or $y = \frac{1}{(x-h)^2}$



Shape 2 – H-Shape (asymptotes form an H)

- This shape has two vertical asymptotes

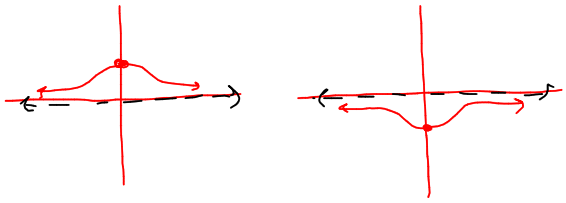
$y = \frac{1}{x^2-k}$



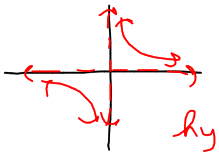
Shape 3 – The Speed Bump or Pot Hole

- This shape has no vertical asymptote

$y = \frac{1}{x^2+k}$



Recall: Linear reciprocal



$y = \frac{1}{x}$

hyperbola

Funnel or Inverted Funnel (One Vertical Asymptote)

Graph $y = \frac{1}{(x-1)^2}$

Step 1: Sketch the graph $y = (x-1)^2$ — parabola

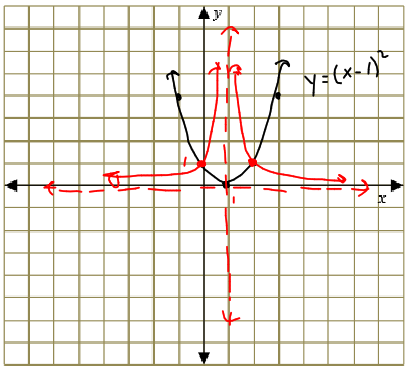
Step 2: Sketch vertical asymptotes at the x-intercepts. This is also the restrictions on the denominator. ($x = 1$, $x \neq 1$, cannot divide by 0)

Note the horizontal asymptote is the x-axis, $y = 0$

Step 3: Plot the invariant points. Where $y = \pm 1$

reciprocal of 1 is 1 and of -1 is -1

Step 4: Sketch the graph, approaching the asymptotes



D: $x \neq 1$

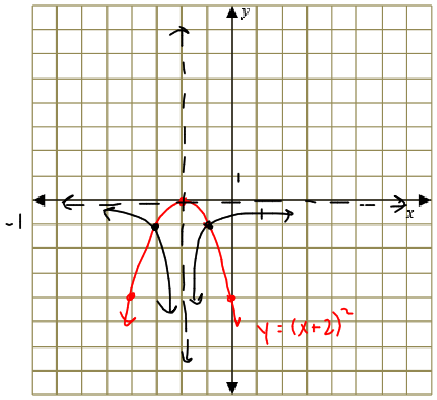
R: $y > 0$ or $(0, \infty)$

Graph $y = \frac{1}{-(x+2)^2}$

$y = -(x+2)^2$

left 2

reflected, opens down



D: $x \neq -2$

R: $y < 0$ or $(-\infty, 0)$

H-Shape (Two Vertical Asymptotes)

Steps:

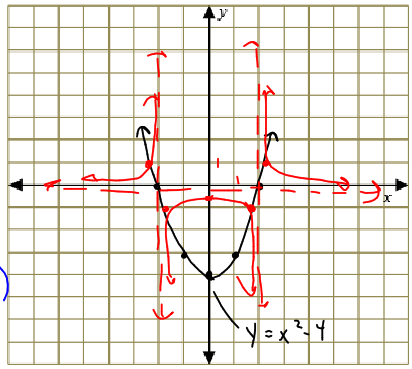
1. Sketch the quadratic function
2. Sketch vertical asymptotes through the x-intercepts
3. Plot the invariant points
4. Take the main points and find their reciprocals
5. Sketch the graph.
6. Remember to erase the original graph or clearly label

min value @ -4
reciprocal is $-\frac{1}{4}$

Graph $y = \frac{1}{x^2-4}$

$y = x^2 - 4$
↑ down 4

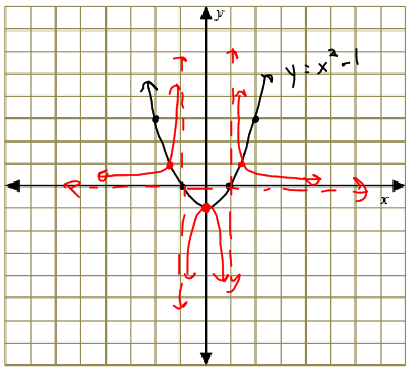
D: $x \neq \pm 2$
* R: $(-\infty, -\frac{1}{4}] \cup (0, \infty)$



Graph $y = \frac{1}{(x-1)(x+1)}$

$y = (x-1)(x+1)$ ← diff of squares
$y = x^2 - 1$

D: $x \neq \pm 1$
R: $(-\infty, -1] \cup (0, \infty)$



Speed Bump or Pot Hole (No Vertical Asymptote)

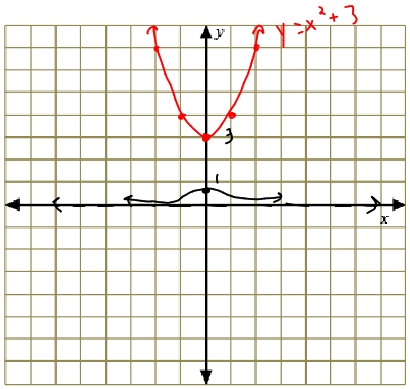
Graph $y = \frac{1}{x^2+3}$

parabola doesn't have x-int(s)

$y = x^2 + 3$

min @ 3

reciprocal $\frac{1}{3}$

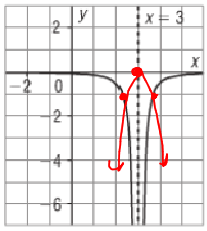


D: $x \in \mathbb{R}$

R: $(0, \frac{1}{3}]$

Use the graph of each reciprocal function $y = \frac{1}{f(x)}$ to graph the quadratic function $y = f(x)$.

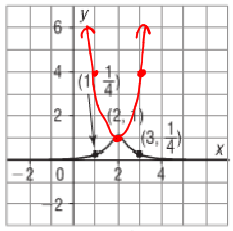
a)



all values on $y = \frac{1}{f(x)}$ are -ve

∴ all values on $y = f(x)$ will be -ve

b)

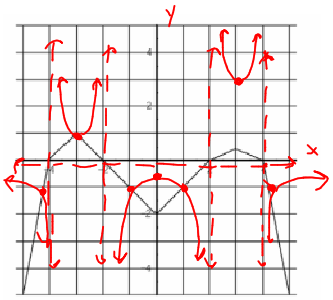


$y = \frac{1}{f(x)}$

reciprocal of $\frac{1}{4}$ is 4

plot these points (1,4) and (3,4)

Draw in the reciprocal of the following graph:



pg. 680 #5, 6, 8a, d 9a, c 11a, b MC #1-3