Pre-Calculus 11 Reciprocals of Linear Functions

Reciprocal Numbers: A reciprocal of any number, x is represented by

$$y = \frac{1}{x}, x \neq 0$$

Reciprocal Functions: A reciprocal of any function, $f(x)$ is represented by

$$y = \frac{1}{f(x)}, f(x) \neq 0$$

Asymptote: a line the graph approaches but never crosses/touches towards infinity or negative infinity.

Determine the reciprocal of each function. Identify values of x for which each rational expression is undefined.

a) $2x$ $\frac{1}{2x}$ $x \neq 0$ → V.A. $x = 0$

b) $x + 4$ $\frac{1}{x+4}$ $x \neq -4$ → V.A. $x = -4$

c) $x^2 - 9$ $\frac{1}{x^2-9}$ $x \neq \pm 3$ → V.A. $x = \pm 3$

$(x-3)(x+3)$

d) $(x - 1)(x + 3)$ $\frac{1}{(x-1)(x+3)}$ $x \neq -3, 1$ → V.A. $x = -3$, $x = 1$

Example 1 – Determine the equation of the vertical asymptote of the graph of $y = \frac{1}{-5x+4}$.

Set denominator equal to 0

Solve

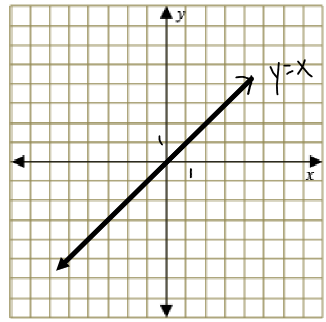
$-5x + 4 = 0$

$4 = 5x$

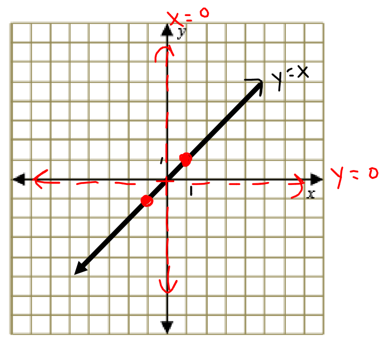
$\frac{4}{5} = x$ ← V.A. @ $x = \frac{4}{5}$

Example 2 – Graph $y = \frac{1}{x}$

Step 1: Draw $y = x$

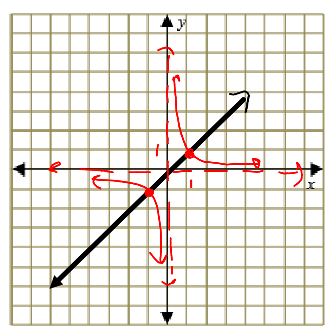


Step 2: ***Vertical asymptote***
This is the restriction of the rational expression. The denominator can't be equal to 0. (goes through x-int(s))
Horizontal asymptote at $y = 0$.



Step 3: Place points wherever $y = \pm 1$ on the original function. These are the points that are same when you take the reciprocal. $1 = \frac{1}{1}$,
and $-1 = \frac{1}{-1}$
These points are called ***invariant points***.

Step 4: Draw in the reciprocal graph.



Remember what happens in basic division of numbers. As the denominator becomes larger, the resulting number becomes smaller.
As the denominator becomes smaller, the resulting number becomes larger.
The graph approaches the asymptotes.

As $f(x)$ approaches 0, $\frac{1}{f(x)}$ approaches ∞ or $-\infty$
and vice versa

Example 3 – Graph the following rational functions.

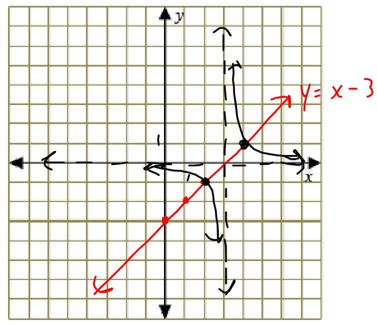
a) $y = \frac{1}{x-3}$

① Sketch y = x - 3

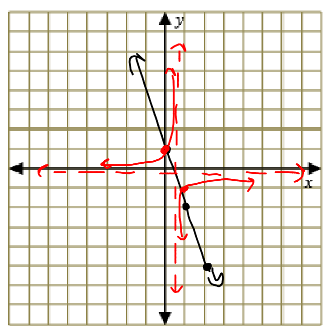
② V.A. through x-int
H.A. @ y = 0

③ Invariant points where
y = 1 and y = -1

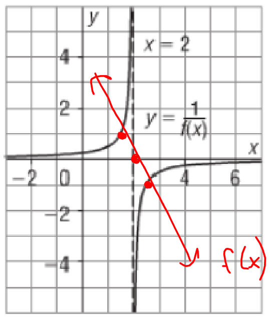
④ Sketch hyperbola shape,
graph approaches the asymptotes



b) $y = \frac{1}{-3x+1}$



Example 4 – Use the graph of $y = \frac{1}{f(x)}$ to graph the linear function $y = f(x)$



① Invariant points at y=1 and y=-1

② Place a point at y=0 on the asymptote (x-int of f(x))

③ Sketch a line through the points.

Assignment: Pg. 657; #3 a, c, 5a, c, 6b, 7b,d, 8a, 11 MC#1-3