

Pre-Calculus 11 Geometric Series

A geometric series is the sum of the terms of a geometric sequence.

1, 3, 9, 27, ... ← geometric sequence

1 + 3 + 9 + 27 + ... ← geometric series

The formula for finding the partial sum of n terms of a geometric series is:

$$S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$$

where: S_n is the sum of n terms
 t_1 is the first term of the series
 r is the common ratio
 n is the number of terms

Examples

- Determine the sum of the first 12 terms of the geometric series:
 $3 + 12 + 48 + 192 + \dots$

$$\begin{aligned} n &= 12 \\ t_1 &= 3 \\ r &= 4 \end{aligned}$$

$$\begin{aligned} S_n &= \frac{t_1(1 - r^n)}{1 - r} \\ S_{12} &= \frac{3(1 - 4^{12})}{1 - 4} \\ &= \frac{\cancel{3}(1 - 4^{12})}{-3} \\ &= -(1 - 4^{12}) \\ &= 16\,777\,215 \end{aligned}$$

2. The sum of the first 14 terms of a geometric series is 16 383. The common ratio is -2. Determine the 1st term.

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

\uparrow
 S_{14}
 $\therefore n=14$

$$16\,383 = \frac{t_1(1-(-2)^{14})}{1-(-2)}$$

$$16\,383 = \frac{t_1(-16\,383)}{3}$$

$$\cancel{16\,383}(3) = t_1(-\cancel{16\,383})$$

$$3 = -t_1$$

$$-3 = t_1$$

3. Calculate the sum of the geometric series: -3 - 15 - 75 - ... - 46 875.

① Determine n

$$t_n = t_1 r^{n-1}$$

$$-46\,875 = -3(5)^{n-1}$$

$$15\,625 = 5^{n-1}$$

$$5^6 = 5^{n-1}$$

$$\therefore 6 = n-1$$

$$7 = n$$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_7 = \frac{-3(1-(5)^7)}{1-5}$$

$$S_7 = -58\,593$$

4. A person takes tablets to cure a chest infection. Each tablet contains 500 mg of an antibiotic. About 15% of the mass of the antibiotic remains in the body when the next tablet is taken. Determine the mass of antibiotic in the body after each number of tablets:

a) 3 tablets

$$S_1 = 500$$

$$S_2 = \underbrace{500(0.15)}_{\substack{\text{15\% of} \\ \text{1st tablet}}} + \underbrace{500}_{\substack{\uparrow \\ \text{new tablet}}}$$

$$\begin{aligned} S_3 &= 500(0.15)(0.15) + 500(0.15) + 500 \\ &= \underbrace{500(0.15)^2}_{\substack{\text{1st tab} \\ \text{2nd tab}}} + \underbrace{500(0.15)}_{\substack{\text{2nd tab} \\ \text{3rd tab}}} + \underbrace{500}_{\substack{\uparrow \\ \text{3rd tab}}} \\ &= 586.25 \text{ mg} \end{aligned}$$

b) 10 tablets

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$\begin{aligned} t_1 &= 500 \\ r &= 0.15 \\ n &= 10 \end{aligned}$$

$$S_{10} = \frac{500(1-0.15^{10})}{1-0.15}$$

$$S_{10} = 588.24 \text{ mg}$$

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3a, 4a
5a, 6b
9a, 10a
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