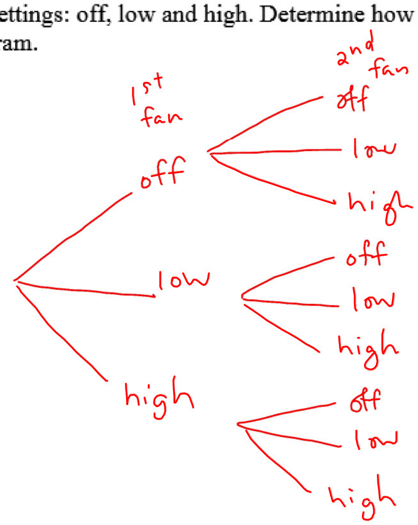
Pre-Calculus 12 Fundamental Counting Principle

Counting methods are used to determine the number of members of a specific set as well as the outcomes of an event. You can display all the possible choices using tables, lists or tree diagrams and then count the number of outcomes.

Ex. 1) A fan has three settings: off, low and high. Determine how many ways are there to set two fans using a tree diagram.



9 branches

$\therefore$ 9 ways

OR

$\underset{\text{1st fan}}{3} \times \underset{\text{2nd fan}}{3} = 9$

Fundamental Counting Principle

If there are n_1 different objects in one set and n_2 different objects in a second set, then the number of ways of choosing one object from each set is $n_1 . n_2$

From example 1,

There are 3 ways to set the first fan and 3 ways to set the second fan so the total number of ways to set 2 fans is $3 \times 3 = 9$

Ex. 2) Determine how many ways a class president, vice-president and secretary can be selected from 30 students to form class council.

$\underset{\text{pres}}{\underline{30}} \cdot \underset{\text{v.p}}{\underline{29}} \cdot \underset{\text{sec}}{\underline{28}} = 24\ 360$ ways

Ex. 3) For an online banking account the minimum security standards require a password to have two letters followed by five digits. All letters and numbers may be used more than once.

repetition is allowed.

a) Determine the number of possible combinations

$\underset{l}{\underline{26}} \cdot \underset{l}{\underline{26}} \cdot \underset{d}{\underline{10}} \cdot \underset{d}{\underline{10}} \cdot \underset{d}{\underline{10}} \cdot \underset{d}{\underline{10}} \cdot \underset{d}{\underline{10}} = 67\ 600\ 000$ ways

0 - 9

b) Determine the number of combinations there would be if repetition is not allowed.

$\underset{l}{\underline{26}} \cdot \underset{l}{\underline{25}} \cdot \underset{d}{\underline{10}} \cdot \underset{d}{\underline{9}} \cdot \underset{d}{\underline{8}} \cdot \underset{d}{\underline{7}} \cdot \underset{d}{\underline{6}} = 19\ 656\ 000$ ways

Ex. 4) The final score in a soccer game is 5 to 4 for Team A. How many different half-time scores are possible?

$\underline{6}_{A} \cdot \underline{5}_{B} = 30$ possible scores

scores 0 - 5

scores 0 - 4

Ex. 5) How many four-digit numbers are there with no repeated digits?

$\underline{9} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} = 4536$

can't be 0 1-9

can be 0, used 1 digit

used 2 digits

Ex. 6) How many odd four-digit numbers have no repeated digits?

last digit must be odd

$\underline{8} \cdot \underline{8} \cdot \underline{7} \cdot \underline{5} = 2240$

(step 2) not 0, not last digit

used 2

start here (step 1) can be 1, 3, 5, 7, 9

Restrictions

Ex. 7) How many different 3-digit even numbers greater than 300 can be made using only the digits 1, 2, 3, 4, 5 and 6? No repetition.

cases

case 1 : ends in 2

$\underset{\text{(step 2) } 3,4,5,6}{\underline{4}} \cdot \underset{\text{used } 2}{\underline{4}} \cdot \underset{2 \text{ (step 1)}}{\underline{1}} = 16$

case 2 : ends in 4 or 6

$\underset{3,4,5 \text{ or } 3,5,6}{\underline{3}} \cdot \underline{4} \cdot \underset{4,6}{\underline{2}} = 24$

add cases

$16 + 24 = 40$ ways

Ex. 8) How many even 4-digit numbers have no repeated digits?

case 1 : ends in 0

have to use cases because of the 0

$\underset{\text{not } 0}{\underline{9}} \cdot \underline{8} \cdot \underline{7} \cdot \underset{0}{\underline{1}} = 504$

case 2 : ends in 2, 4, 6, 8

$\underset{\text{not } 0, \text{ used } 1}{\underline{8}} \cdot \underline{8} \cdot \underline{7} \cdot \underset{2,4,6,8}{\underline{4}} = 1792$

Total = 504 + 1792
2296 ways

Assignment: Pg. ~~688, #5-6, 8-9, 16, MC #1-3, Worksheet for extra practice~~