Pre-Calculus 12
Financial Application of Logs

Future Value

When a series of equal investments is made at equal time intervals, and the compounding period for the interest is equal to the time interval for the investments, the amount in dollars, or Future value (FV), of these investments can be determined using the formula:

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

future value

R = amount of investment

i = interest rate

# of compounding periods

n = number of investments

Example

Determine how many monthly investments of $200 would have to be made into an account that pays 6% annual interest, compounded monthly, for the future value to be $100 000.

$R = 200$

$i = \frac{0.06}{12}$ or 0.005

$n = ?$

$FV = \$100000$

$$100\,000 = \frac{200[(1+0.005)^n - 1]}{0.005}$$

mult

$$\frac{500}{200} = \frac{200}{200}[(1+0.005)^n - 1]$$

$$2.5 + 1 = (1+0.005)^n - 1 + 1$$

$$3.5 = (1+0.005)^n$$

$$3.5 = 1.005^n$$

$$\log 3.5 = n \log 1.005$$

$$\frac{\log 3.5}{\log 1.005} = n$$

$$251.178 = n$$

or 252 payments

Present Value

Many people borrow money to finance a purchase. A loan is usually repaid by making regular equal payments for a fixed period of time. The amount borrowed is called the Present value (PV), of the loan.

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

R = amount of payment

i = interest rate
# of compound periods

n = number of payments

Example

A person borrows $15 000 (PV) to buy a car. The person can afford to pay $300 (R) a month. The loan will be repaid with equal monthly payments at 6% annual interest, compounded monthly. How many monthly payments will the person make?

$PV = 15\,000$

$R = 300$

$i = \frac{0.06}{12}$

$n = ?$

$15\,000 = \frac{300[1-(1+0.005)^{-n}]}{0.005}$

mult

$\frac{75}{300} = \frac{300[1-(1.005)^{-n}]}{300}$

$0.25 = 1-(1.005)^{-n}$

$-0.75 = -(1.005)^{-n}$

$0.75 = 1.005^{-n}$

$\log 0.75 = -n \log 1.005$

$\frac{\log 0.75}{(-\log 1.005)} = n$

58 payments $= n$

Review
pg 416
Practice Test
pg 419

Compound Interest

Compound Interest: earning interest on interest, can be computed annually, semi-annually, quarterly, monthly

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A A_0

A = amount accumulated after t years

P = amount invested

r = annual interest rate

t = time in years

n = number of compounding periods per year

Examples

1. A \$5000 investment earns interest at an annual rate of 8.4% compounded monthly.

   a) What is the investment worth after 10 years?

   $$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
   $$= 5000\left(1 + \frac{0.084}{12}\right)^{12(10)}$$
   $$= \$11547.99$$

   b) How much interest was earned?

   Interest = \$11547.99 − \$5000

   = \$6547.99

2. A principal of \$1 500 is invested at 4% annual interest, compounded quarterly. To the nearest quarter of a year, when will the amount be \$2 500?

$n = 4$

$\frac{25}{15}$

$\frac{5}{3}$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$2500 = 1500\left(1 + \frac{0.04}{4}\right)^{4t}$$

$$\frac{5}{3} = (1.01)^{4t}$$

$$\log \frac{5}{3} = 4t \log 1.01$$

$$\frac{\log \frac{5}{3}}{(4 \log 1.01)} = t$$

$$12.834 = t$$

13 yrs

3. Find the time required for an investment of \$100 to double at an annual rate of 8% compounded semi-annually.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$200 = 100\left(1 + \frac{0.08}{2}\right)^{2t}$$

$$2 = 1.04^{2t}$$

$$\log 2 = 2t \log 1.04$$

$$\frac{\log 2}{(2 \log 1.04)} = t$$

8.836 yr = t

or 9 yrs

pg. 435
\# 3, 6, 7, 9, 12

Worksheet
b