Pre-Calculus 12 Factor Theorem

Theorem: A polynomial $P(x)$ has a factor $x-a$ iff $P(a)=0$
(if and only if)

ie. if the remainder is 0, $x-a$ is a factor of $P(x)$

Ex .1) Factor: $f(x) = x^3 - 2x^2 - 5x + 6$

$$possible\ "a"\ values = \frac{factors\ of\ the\ constant\ term}{factors\ of\ the\ leading\ coefficient}$$

poss "a" values $= \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1}$ ← factors of 6

$= \pm 1, \pm 2, \pm 3, \pm 6$

Steps:

1. List all possible factors
2. Begin evaluating P(x) using the above factors
3. Stop when you find one that equals 0
4. Divide
5. Factor completely

$f(2) = 2^3 - 2(2)^2 - 5(2) + 6$
$= 8 - 8 - 10 + 6$
$= -4$ ✗ remainder is not 0 ∴ $x-2$ is not a factor

$f(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$
$= -8 - 8 + 10 + 6$
$= 0$ ✓
∴ $x+2$ is a factor

-2	1	-2	-5	6
		-2	8	-6
	1	-4	3	0

quot $x^2 - 4x + 3$

$f(x) = (x+2)(x^2 - 4x + 3)$
$f(x) = \underbrace{(x+2)(x-3)(x-1)}_{\text{factors of } f(x)}$

P 3
S -4
F -3, -1

Ex. 2) Solve.

$$3x^3 - 4x^2 - 5x + 2 = 0$$

← what values make this eqn true? ie roots

poss "a" values = $\frac{\text{factors of } 2}{\text{factors of } 3}$

$= \frac{\pm 1, \pm 2}{\pm 1, \pm 3}$

$= \pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$

$f(2) = 3(2)^3 - 4(2)^2 - 5(2) + 2$

$= 24 - 16 - 10 + 2$

$= 0$ ✓

∴ $x - 2$ is a factor

2	3	−4	−5	2
	↓	6	4	−2
	3	2	−1	0

quot $3x^2 + 2x - 1$

$(x-2)(3x^2+2x-1) = 0$

$(x-2)(3x-1)(x+1) = 0$

solve

$x - 2 = 0$ → $x = 2$

$3x - 1 = 0$ → $3x = 1$ → $x = \frac{1}{3}$

$x + 1 = 0$ → $x = -1$

P −3
S 2
F $\frac{3}{3}, \frac{-1}{1}$

Bulawka's Bullets

☺ **Know the difference between factor and solve**

☺ **The roots of the equation are the same as the x-intercepts of the graph**

Assignment: Pg ~~22; #9, 10, 11, 12~~ pg 133 #1, 2c, e, 5a, c, 6b, 11